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IMAGE FUSION ON THE BASIS OF FUZZY TRANSFORMS

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We show that on the basis of fuzzy transform, the problem of reconstruction of corrupted images can be solved. The proposed technique is called image fusion. An algorithm of image fusion, based on fuzzy transform, is proposed and justified. A measure of fuzziness of an image is proposed as well.

Keywords: Image Fusion; Fuzzy partition; Fuzzy transform.

1. Introduction

The fuzzy transform proves to be a successful methodology with various applications from image compression and reconstruction to numeric solution of differential equations ([1,2]). In this contribution the technique of fuzzy transforms (F-transforms) is applied to image fusion.

The structure of the contribution is the following. In Section 2, the technique of the direct and inverse fuzzy transform is recalled. In Section 3, the problem of image fusion is introduced and the algorithm, based on fuzzy transform, is proposed and justified. The illustration is given at the end.

2. Fuzzy transform

In this section, we recall ([1,2]) the notion of fuzzy transform $\mathbf{F}_n[f]$ of a continuous function f on $[a, b]$. We will further use the short name F-transform instead of fuzzy transform. We will also recall some approximation properties of the both direct and inverse F-transforms.

2.1. *Fuzzy Partition of the Universe*

The key idea of the fuzzy transform is a fuzzy partition of a universe into fuzzy subsets (factors, clusters, granules etc. as propagated by L.A. Zadeh).

An interval $[a, b]$ is considered as a universe. That is, all (real-valued) functions considered in this contribution have this interval as a common domain.

Definition 2.1. Let $x_1 < \ldots < x_n$ be fixed nodes within $[a, b]$, such that $x_1 = a$, $x_n = b$ and $n \geq 3$. Fuzzy sets $A_1, \ldots, A_n$, identified with their membership functions $A_1(x), \ldots, A_n(x)$ defined on $[a, b]$, form a *fuzzy partition* of $[a, b]$ if for $k = 1, \ldots, n$

- $A_k : [a, b] \longrightarrow [0, 1]$, $A_k(x_k) = 1$;
- $A_k(x) = 0$ if $x \notin (x_{k-1}, x_{k+1})$ where for the uniformity of denotation, we put $x_0 = a$ and $x_{n+1} = b$;
- $A_k(x)$ is continuous;
- $A_k(x)$, $k = 2, \ldots, n$, strictly increases on $[x_{k-1}, x_k]$ and $A_k(x)$, $k = 1, \ldots, n-1$, strictly decreases on $[x_k, x_{k+1}]$;
- for all $x \in [a, b]$, $\sum_{k=1}^{n} A_k(x) = 1$.

The membership functions $A_1, \ldots, A_n$ are called *basic functions*.

Let us remark that the shape of basic functions is not predetermined and therefore, it can be chosen additionally.

The fuzzy partition is *uniform* if the nodes $x_1, \ldots, x_n$, $n \geq 3$, are equidistant.

2.2. *Direct fuzzy transform*

Let us fix an interval $[a, b]$ and nodes $x_1 < \ldots < x_n$, such that $x_1 = a$, $x_n = b$ and $n \geq 3$. Let $A_1, \ldots, A_n$ be some fixed basic functions which constitute a fuzzy partition of $[a, b]$. Denote $C([a, b])$ the set of continuous functions on the interval $[a, b]$.

Definition 2.2. Let $A_1, \ldots, A_n$ be basic functions which constitute a fuzzy partition of $[a, b]$ and f be any function from $C([a, b])$. The n-tuple of real numbers $[F_1, \ldots, F_n]$ given by

$$F_k = \frac{\int_a^b f(x) A_k(x) dx}{\int_a^b A_k(x) dx}, \qquad k = 1, \ldots, n,$$

is the (direct) F-transform of f with respect to $A_1, \ldots, A_n$. The elements $F_1, \ldots, F_n$ are called *components of the F-transform*.

Denotation: $\mathbf{F}_n[f] = [F_1, \ldots, F_n]$.

At this point we will refer to[1,2] for the full list of up to date known properties of the F-transform. Below we will repeat only those properties which are relevant to the proposed technique.

Lemma 2.1. *Let f be a continuous function on $[a, b]$ and $A_1, \ldots, A_n$, $n \geq 3$, be basic functions which constitute a uniform fuzzy partition of $[a, b]$. Let $F_1, \ldots, Fn$, be the F-transform components of f with respect to $A_1, \ldots, A_n$. Then for each $k = 1, \ldots, n-1$, and for each $t \in [x_k, x_{k+1}]$ the following estimations hold:*

$$|f(t) - F_k| \leq 2\omega(h, f), \qquad |f(t) - F_{k+1}| \leq 2\omega(h, f)$$

where $h = \frac{b-a}{n-1}$ and

$$\omega(h, f) = \max_{|\delta| \leq h} \max_{x \in [a, b-\delta]} |f(x+\delta) - f(x)|$$

is the modulus of continuity of f on $[a, b]$.

Theorem 2.1. *Let f be a continuous function on $[a, b]$ and $A_1, \ldots, A_n$ be basic functions which constitute a fuzzy partition of $[a, b]$. Then the k-th component F_k $(k = 1, \ldots, n)$ minimizes the function*

$$\Phi(y) = \int_a^b (f(x) - y)^2 A_k(x) dx.$$

Lemma 2.2. *Let function f be a continuous on $[a, b]$ and basic functions $A_1, \ldots, A_n$, $n \geq 3$, constitute a uniform fuzzy partition of $[a, b]$. Let $F_1, \ldots, F_n$, be the F-transform components of f with respect to $A_1, \ldots, A_n$. Then*

$$\int_a^b f(x) dx = h(\frac{1}{2}F_1 + F_2 + \cdots + F_{n-1} + \frac{1}{2}F_n).$$

2.3. *Inverse F-transform*

The inverse F-transform is given by the inversion formula and approximates the original function in such a way that a universal convergence can be established.

Definition 2.3. Let $A_1, \ldots, A_n$ be basic functions which form a fuzzy partition of $[a, b]$ and f be a function from $C([a, b])$. Let $\mathbf{F}_n[f] = [F_1, \ldots, F_n]$ be the F-transform of f with respect to $A_1, \ldots, A_n$. Then the function

$$f_{F,n}(x) = \sum_{k=1}^{n} F_k A_k(x) \tag{1}$$

is called *the inverse F-transform.*

The theorem below shows that the inverse F-transform $f_{F,n}$ can approximate the original continuous function f with an arbitrary precision. The proof can be found in.[1,2]

Theorem 2.2. *Let f be a continuous function on $[a, b]$. Then for any h-uniform fuzzy partition $A_1, \ldots, A_n$ of $[a, b]$ (where $n \geq 3$ and $h = \frac{b-a}{n-1}$) and for all $x \in [a, b]$,*

$$|f(t) - f_{F,n}(t)| \leq 2\omega(h, f) \tag{2}$$

where $f_{F,n}$ is the inverse F-transform of f with respect to the fuzzy partition $A_1, \ldots, A_n$.

3. Application to Image Fusion

Image fusion aims at integration complementary multiview information into one new image with the best possible quality. The term "quality" depends on the demands of specific application.

Mathematically, if u is an ideal image (considered as a function with two variables) and $C_1, \ldots, C_N$ are acquired channels then the relation between each C_i and u is expressed by

$$C_i(x, y) = D_i(u(x, y)) + E_i(x, y)$$

where D_i is an unknown operator describing the image degradation and E_i is an additive random noise. To fuse images from channels means to obtain an image $\hat{u}$ which gives in some sense better representation of u than each individual channel C_i. Different fusion methodologies are influenced by peculiarities of degradation operators D_i. In this contribution, we assume that every point (x, y) of the image can be acquired undistorted in (at least) one channel. Image fusion then consists of comparing the channels in image domain, identifying the channel in which the pixel (or the region) is depicted undistorted and, finally, of combining the undistorted parts.

To find the fused image, we propose the following algorithm (for the simplicity we assume that all images are represented by functions with one variable on the same domain $[a, b]$ and a uniform fuzzy partition of $[a, b]$ is fully determined by the number of basic functions):

Step 1. Choose $\varepsilon > 0$ and let $k = 1$. Denote each channel function C_i by f_i^k, $i = 1, \ldots, N$.

Step 2. Let $n = 2^k$. Compute the direct and inverse fuzzy transforms of all N functions f_i^k. Denote the direct fuzzy transforms $\mathbf{F}_n[f_i^k]$ and the inverse fuzzy transforms $f_{F,n,i}^k$, $i = 1, \ldots, N$.

Step 3. Compute differences and denote $f_i^{k+1} = f_i^k - f_{F,n,i}^k$, $i = 1, \ldots, N$. If $\max_{[a,b]} |f_i^{k+1}| > \varepsilon$, let $k = k + 1$ and go to *Step 2*. Otherwise, denote $k_{max} = k$.

Step 4. Compute the new "sharp" direct fuzzy transform $\mathbf{F}_n[s^k] = [S_1^k, \ldots, S_n^k]$ where for each $j = 1, \ldots, n$, S_j^k is that value among $\mathbf{F}_n[f_i^k]_j$ whose absolute value is the largest.

Step 5. Compute the new "sharp" inverse fuzzy transform s^k with the components given by $\mathbf{F}_n[s^k]$.

Step 6. Let $k = k - 1$ and until $k \geq 1$ repeat *Step 4.* and *Step 5.*

Step 7. Compute $\sum_{k=1}^{k_{max}} s^k$ and take it as a fused image.

Justification. By Lemma 2.1, the smaller the modulus of continuity the higher the quality of approximation of an original function by its inverse fuzzy transform. Therefore, for the same partition the accuracy of approximation of each difference function f_i^k by its inverse fuzzy transform is better than the accuracy of approximation of the original function C_i by its inverse fuzzy transform.

If a certain part of a function is affected by degradation, then by Theorem 2.1, the respective fuzzy transform component is close to zero. Therefore, on *Step 4.* we chose components with maximal absolute values.

On Figure 1 we show two-channel image fusion based on F-transform technique: In one channel, the figure is in focus and the background is out of focus, while in the other channel it is vice versa. Image fusion is performed via combining the channel regions which are in focus.

Finally, the following value is proposed as a measure of degradation of each channel $i = 1, \ldots, N$:

$$M_i^{deg} = \sum_{k=2}^{k_{max}} \sum_{j=1}^{2^k} |\mathbf{F}_n[f_i^k]_j|.$$

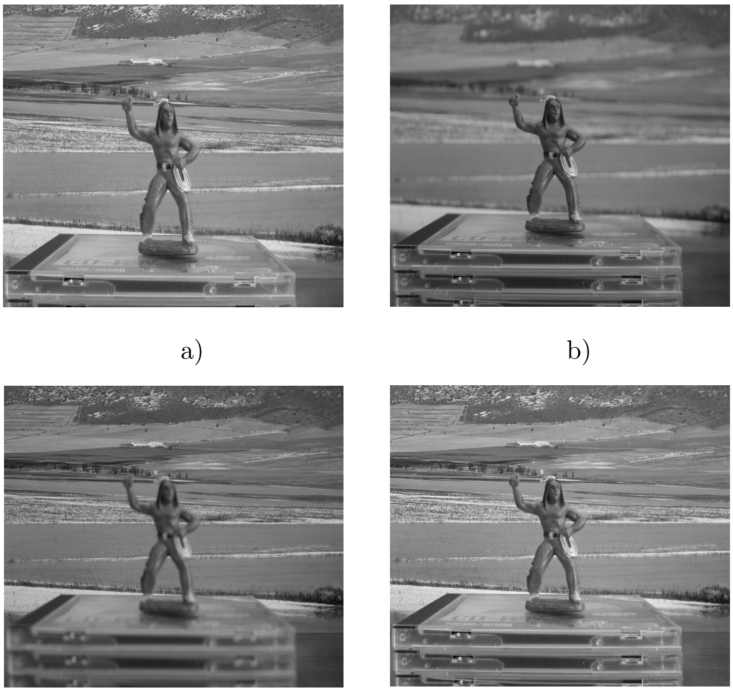
By Lemma 2.2, we obtain the following estimation

$$M_i^{deg} \leq \sum_{k=3}^{k_{max}} \int_a^b |f_i^{k-1}(x) - f_i^{k-2}(x)| dx.$$

Therefore, the bigger M_i^{deg} the less degradation has the channel C_i.

4. Conclusion

An application of fuzzy transform to image processing has been presented. An algorithm of image fusion, based on fuzzy transform, was proposed and



a) b)

c) d)

Fig. 1. The original image is given on Picture a)). Two channel images: Picture b) - the figure is in focus) and Picture c)- the background is in focus) are fused with the result on Picture d).

justified. A measure of fuzziness of an image was proposed on the basis of the algorithm.

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