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Higher degree fuzzy transform

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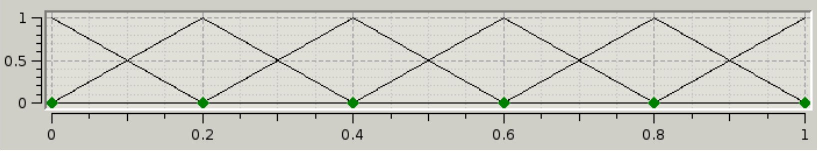
Outline

Fuzzy Partition



Ruspini condition: $\sum_{i=1}^{n} A_i(x) = 1$

Fuzzy transform

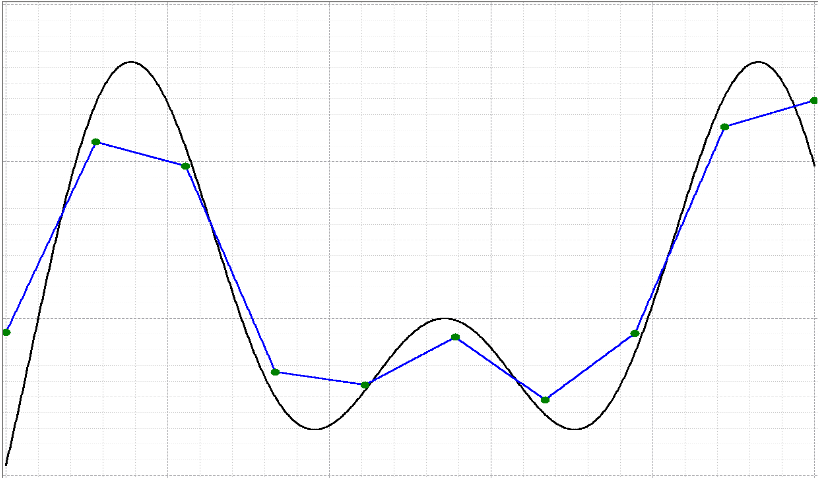
Direct F-transform

$$F_k = \frac{\int_a^b f(x)A_k(x)dx}{\int_a^b A_k(x)dx}, \quad k = 1, \ldots, n.$$

Inverse F-transform

$$f_{F,n}(x) = \sum_{i=1}^{n} F_i A_i(x), \quad x \in [a, b]$$

sin(3x)+cos(2x), 10 components



Outline

F^m-transform

Be smoother and differentiable

- Do not approximate by constants but use polynoms:

$$F_i(x) = \sum_{j=0}^{m} F_{ij}(x - x_i)^j$$

- Approximate m-th derivation of $f(x)$
- Better shape of approximation than just integral equality

IRAFM

Hilbert space L_2

Theoretical background, construction for each $A_k(x)$

- $L_2(A_k)$ set of square-integrable functions $f, g : [a, b] \to \mathbb{R}$

$$\langle f, g \rangle = \int_{x_{k-1}}^{x_{k+1}} f(x)g(x)A_k(x)dx$$

- $L_2^m(A_k)$ be a subspace of polynomials of $L_2(A_k)$ with the orthogonal base $< P^0, P^1, \dots, P^m >$

IRAFM

Orthogonalization

Recursive process

- Take polynomials: $1, x, x^2, x^3, \dots, x^m$
- Apply Gram-Schmidt process

$$P_k = 1,\ P_k^{r+1} = x^{r+1} - \sum_{i=0}^{r} \frac{\langle x^{r+1}, P_k^i \rangle}{\langle P_k^i, P_k^i \rangle} P_k^i,\ r = 0, \dots, m-1$$

Polynomial base

$P^0 = 1$
$P^1 = x$
$P^2 = x^2 - \langle x^2, 1 \rangle \simeq x^2 - \frac{h^2}{6}$
$P^3 = x^3 - \frac{\langle x^4, 1 \rangle}{\langle x^2, 1 \rangle} x \simeq x^3 - \frac{2h^2}{5} x$

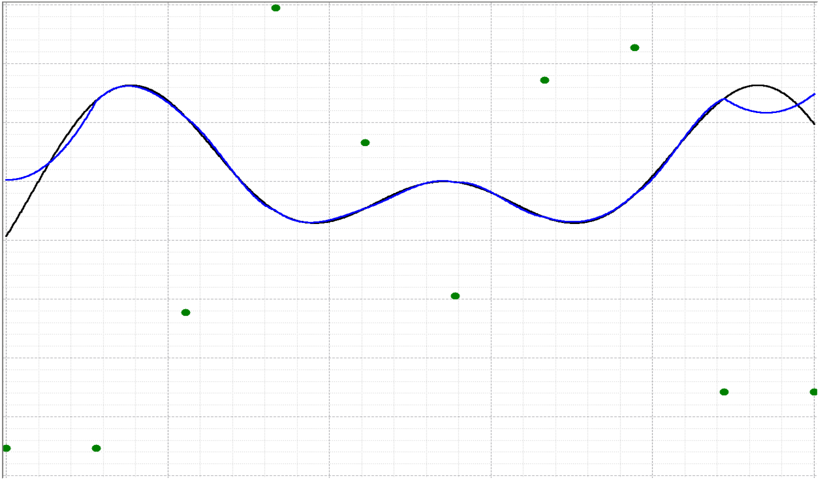
Orthogonal projection

- Let's call a n-tuple $(F_1^m, \ldots, F_n^m)$ as an F^m-transform
- $F_k^m = c_{k,0}P^0 + c_{k,1}P^1 + \ldots + c_{k,m}P^m$

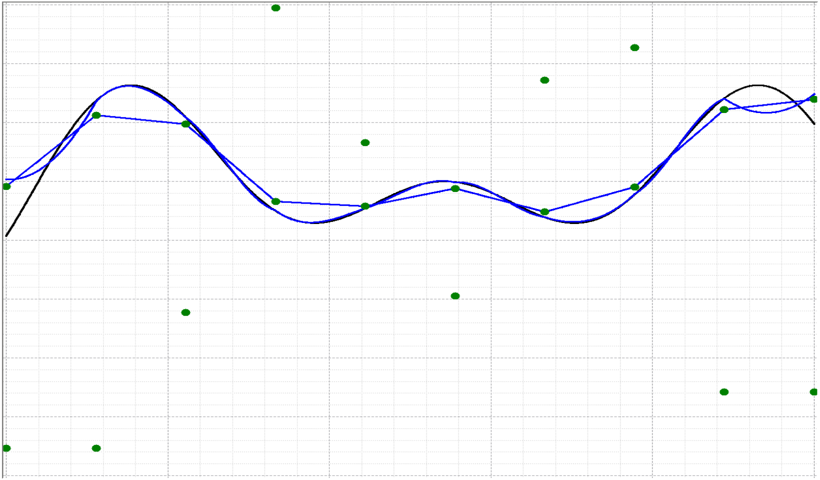
$$c_{k,i} = \frac{\langle f, P_k^i \rangle}{\langle P_k^i, P_k^i \rangle} = \frac{\int\limits_a^b f(x)P_k^i(x)A_k(x)dx}{\int\limits_a^b P_k^i(x)P_k^i(x)A_k(x)dx}, \quad i = 0, \ldots, m$$

- Inverse F-transform is same formula

sin(3x)+cos(2x), 10 components, F^2



sin(3x)+cos(2x), 10 components, both



Outline

Conclusion

Pros

- Better approximation of $f(x)$ with higher degree
- Approximate also derivatives

Cons

- Computation is more complex and time consuming

Thanksgiving

printf("Thank You for Your attention");