Small subgraphs of triangle-free *SRG*

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- Strongly regular graphs.
- Induced subgraphs of *SRG* of order at most 5.
- Subgraph as a parameter.
- The solution for parametric cases.

Definition

Strongly regular graph (SRG) with the parameters (n, k, λ, μ) is a k-regular graph on n vertices with following properties:

1. Any two adjacent vertices have exactly λ common neighbours.
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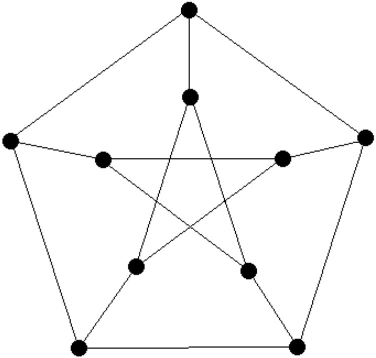

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- *There are constructions which are specialised for concrete SRG.*
- *Some known constructions are usable for whole classes of SRG.*
- *Some properties necessary for the existance of SRG are known.*
- *There are still many SRG about which we know nothing.*

SRG(n, k, 0, 1) (Moore graphs)

Table: Hoffman and Singleton, 1960

k	n	
2	5	pentagon
3	10	Petersen graph
7	50	Hoffman-Singleton graph
57	3250	?

SRG(n, k, 0, 2)

k	n	
2	4	4-cycle
5	16	Clebsch graph
10	56	Sims-Gewirtz graph
26	356	?
37	704	?
⋮	⋮	?

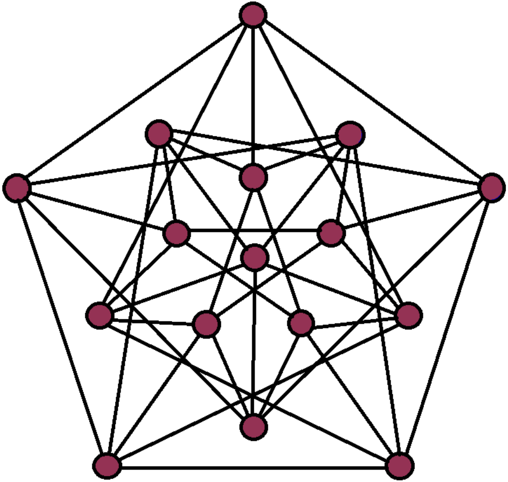

Figure: Clebsch graph, $SRG(16, 5, 0, 2)$

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Remark

It is possible to consider the number of occurrence of some induced subgraph in SRG as its new characterization.

Proposition

We are able to determine the number of occurrrnces of a graph G as induced an subgraph in $SRG(n, k, 0, \mu)$ *using only the paramethers* n, k, μ.

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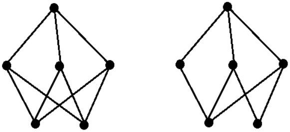
Remark

1. *Let G be a graph of order k and v be one of its vertices. Then occurrences of G in SRG depends on the number of occurrence of the graph $G - v$.*
2. *There exist graphs whose number of occurrance in $SRG(n, k, 0, \mu)$ is not a constant. Graphs of this type will be called parameters of SRG. (If $\mu = 3$ then $K_{3,3}$ is a parameter.)*



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Let v be some vertex of the graph G.
We derive the number of occurrence of induced subgraph $G(V, E)$ in SRG. For this goal we use the graph $G - v$, where v satisfies the following properties:

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- $\deg(v) = \min_{u \in V} \deg(u) = \delta(G)$
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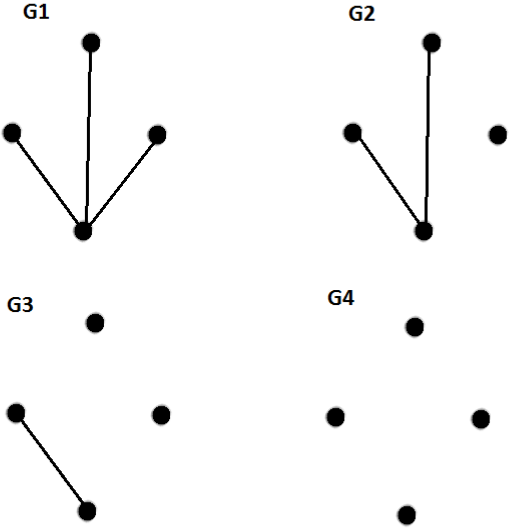
If the graph G is the parameter of SRG, the graph $G - v$ has to contain the independant set of order 3.


G1
G2
G3
G4

Let G be a constant of $SRG(n, k, 0, \mu)$. Then for the number (n_G) of its occurrence in SRG holds:

$$n_G = \frac{o(S_v).|G - v|.x}{o(v)},$$

where:

- S_v is the set of all neigbours of the vertex v in the graph G,
- $o(S_v)$ denotes the orbit of the vertex set S_v with respect to the group $Aut(G - v)$,
- $|G - v|$ is the number of occurrences of $G - v$ in SRG,
- $o(v)$ is the orbit of v with respect to the group $Aut(G)$,
- x represents the number of different vertices in SRG, whose set of neighbours in $G - v$ is exactly S_v.

A relation between the parameters of $SRG(n, k, 0, \mu)$ we obtain from the solution of the following system of equations:

$$|G|.(n - |V|) = \sum_{S \subseteq V} |G_S|,$$

where $|G|$ denotes the number of induced subgraphs of the shape G.
A graph G_S comes from the graph G by adding one new vertex whose neighbour set will be S.
$|G_S|$ represents the number of induced subgraphs of the shape G_S in SRG.

Special case

Between induced subgraph of order 6 in $SRG(n, k, 0, \mu)$, where $\mu > 2$, there is the only parameter ($K_{3,3}$).
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Goal

We would like to use this method for describing subgraphs of order at most 10.
(An example of a subgraph of order 10 *is the Petersen graph.)*

Thank you for attention ☺