AHP in evaluation of creative work outcomes of Czech art colleges

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Key topics

- Register of Artistic Performances (RAP)

- Classification of Works of Art

- Determining scores for particular categories
  of artistic production (mathematical model)

Register of Artistic Performances (RAP) (RUV in Czech)

- Information on works of art (of colleges) + „evaluation"
- Already being pilot tested
- Development continues
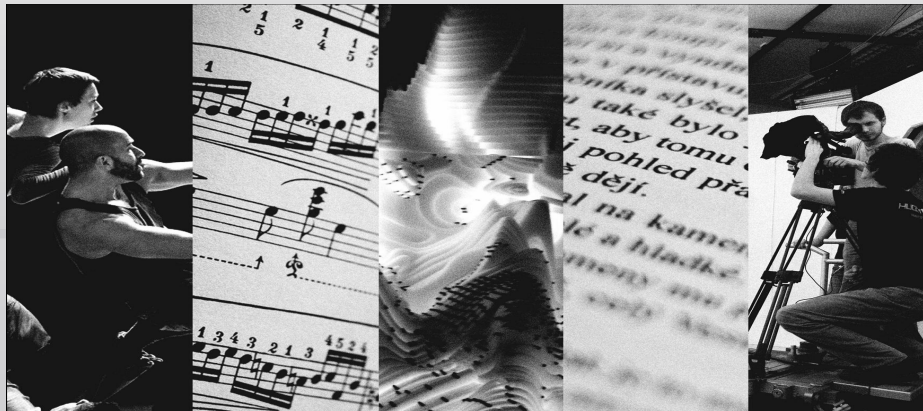
(an analogy to the register of R&D outcomes (**RIV**) in CZ)

?Preview?

A little preview

a short video characterising the purpose of RAP

Register of Artistic Performances (RAP) (RUV in Czech)

- Creative work outcomes are stored there under **several categories**.

- Categories are assigned **scores**.

- **Performance indicator** - sum of scores of all the outcomes of a given art college.

- Decisions about **allocation money** from the state budget among the art colleges.

Classification of Works of Art

- The whole area of artistic production is divided into 7 fields:

  fine arts, design, architecture, theatre, film, literature, music

- Pieces of art categorized according to the following criteria:

  - **relevance or significance**

  - **extent**

  - **institutional and media reception/impact**

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Classification of Works of Art (criteria, levels)

- **Relevance or significance of the piece of art (A,B,C,D)**:
  - **A** – crucial significance;
  - **B** – numerous important innovations;
  - **C** – pushing up modern trends.
- **Extent of the piece of art**:
  - **K** - large extent;
  - **L** - medium extent;
  - **M** - limited extent.
- **Institutional and media reception/impact of the piece of art**:
  - **X** – international impact,
  - **Y** – national impact,
  - **Z** – regional impact.

The resulting **category for a piece of art** is given by a combination of three capital letters – e.g. AKX, BKY, or CLZ.

Determining scores for particular categories of artistic production

- **Interactions among** the three **criteria**

- Approach:

  1. setting weights of criteria

  2. setting scores for each of them

  3. calculating the scores of categories by means of WA.

  ## NOT POSSIBLE

- We have **set directly the scores of the categories** described by the triples of criteria levels.

  (**Saaty's method** - scores for all 27 categories)

Determining scores for particular categories of artistic production

Two-step procedure for converting expert prefeneces into scores:

1) Pair-wise comparison method

⇨

order of importance of the categories

2) Saaty's matrix of preference intensities

⇨

scores (consistency weakened)

The Pairwise Comparison Method

- The method employs a **matrix P of preferences and indifferences**

$$P = \left\{ p_{i,j} \right\}_{i,j=1,\ldots,27},$$

$p_{ij}=1$ …… the **i**th category is **more important** than the **j**th category;

$p_{ij}=0.5$ …. the **i**th category is **equally important** as the **j**th category;

$p_{ij}=0$ …… the **j**th category is **more important** than the **i**th category.

- We need to verify **consistency** of the preferences in the sense of transitivity:

$$p_{i,k} \geq \max\left\{ p_{i,.j}, p_{j,k} \right\} \quad \text{for all } i, j, k \in \{1,\ldots,27\}$$

- If the matrix is **not consistent**, we make a minimum amount of changes necessary for it to become so (changes are then consulted with the team of experts)

The Pairwise Comparison Method – necessary changes

		1 AKX	2 AKY	3 AKZ	4 ALX	5 ALY	6 ALZ	7 AMX	8 AMY	9 AMZ	10 BKX	11 BKY	12 BKZ	13 BLX	14 BLY	15 BLZ	16 BMX	17 BMY	18 BMZ	19 CKX	20 CKY	21 CKZ	22 CLX	23 CLY	24 CLZ	25 CMX	26 CMY	27 CMZ	Preference	Preference order	
1	AKX	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	26,5	1	AKX
2	AKY		0,5	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	25	2	AKY
3	AKZ			0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	25	2	AKZ
4	ALX				0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	23,5	4	ALX
5	ALY					0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	21,5	6	ALY
6	ALZ						0,5	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	20,5	7	ALZ
7	AMX							0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	22,5	5	AMX
8	AMY								0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	18,5	9	AMY
9	AMZ									0,5	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	17,5	10	AMZ
10	BKX										0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	19,5	8	BKX
11	BKY											0,5	1,0	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	16	11	BKY
12	BKZ												0,5	0,0	1,0	1,0	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	14	13	BKZ
13	BLX													0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	16	11	BLX
14	BLY														0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	12,5	15	BLY
15	BLZ															0,5	0,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	11,5	16	BLZ
16	BMX																0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	14	13	BMX
17	BMY																	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	10,5	17	BMY
18	BMZ																		0,5	0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	9	18	BMZ
19	CKX																			0,5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	9	18	CKX
20	CKY																				0,5	1,0	0,0	1,0	1,0	1,0	1,0	1,0	6,5	21	CKY
21	CKZ																					0,5	0,0	1,0	1,0	1,0	1,0	1,0	5,5	22	CKZ
22	CLX																						0,5	1,0	1,0	1,0	1,0	1,0	7,5	20	CLX
23	CLY																							0,5	1,0	0,0	1,0	1,0	3,5	24	CLY
24	CLZ																								0,5	0,0	1,0	1,0	2,5	25	CLZ
25	CMX																									0,5	1,0	1,0	4,5	23	CMX
26	CMY																										0,5	1,0	1,5	26	CMY
27	CMZ																											0,5	0,5	27	CMZ

changes made to achieve consistency of the matrix resulting form the final order

change made from 0,5 to 1 or from 1 to 0,5 ("small change")

change made from 0 to 1 ("big change")

The Saaty's Method

- **Saaty's matrix S of preference intensities** (categories numbered in accordance with their significance):

$$S = \left\{ s_{i,j} \right\}_{i,j=1,\ldots,27}$$

The elements $s_{ij}, i < j$, were set as follows:

$s_{ij}=1$ … the ith and j^{th} categories are **equally** important;

$s_{ij}=3$ … the ith category is **slightly more important** than the jth category;

$s_{ij}=5$ … the ith category is **strongly more important** than the jth category;

$s_{ij}=7$ … the ith category is **very strongly more important** than the jth category;

$s_{ij}=9$ … the ith category is **extremely more important** than the jth category.

It holds $s_{ii}=1$ and $s_{ij}=1/s_{ji}$.
s_{ij} expresses the ratio of preferences between both categories.

The Saaty's Method

- The traditional requirement for **consistency** in Saaty's method, that is

$$s_{ij}=s_{ik}.s_{kj}$$ for all i,j,k $\in \{1,\ldots,27\}$,

  is **basically unachievable**.

- We have **weakened the original requirement on consistency**:

$$s_{i,k} \geq \max\{s_{i,.j}, s_{j,k}\}$$ for all i,j,k $\in \{1,\ldots,27\}$.

- When the categories are numbered as to their importance, this requirement means that the elements of S are **nondecreasing from left to right and from bottom up.**

- If the matrix, as set by the experts, is not consistent, we propose **the minimum amount of changes necessary** for it to become so.

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The Saaty's Method – necessary changes

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
		AKX	AKY	AKZ	ALX	AMX	ALY	ALZ	BKX	AMY	AMZ	BKY	BKZ	BLX	BMX	BLY	BLZ	BMY	BMZ	CKX	CLX	CKY	CKZ	CMX	CLY	CLZ	CMY	CMZ
1	AKX	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	9	9	9	9	9	9	9	9	9	9
2	AKY		1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	7	9	9	9	9	9	9
3	AKZ			1	3	3	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	7	9	9	9	9	9	9
4	ALX				1	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9	9	9	9	9
5	AMX					1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9	9	9
6	ALY						1	3	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9	9
7	ALZ							1	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7	9
8	BKX								1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
9	AMY									1	3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
10	AMZ										1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
11	BKY											1	5	5	5	5	5	5	5	5	5	5	5	5	5	7	7	7
12	BKZ												1	3	5	5	5	5	5	5	5	5	5	5	5	7	7	7
13	BLX													1	5	5	5	5	5	5	5	5	5	5	5	7	7	7
14	BMX														1	3	3	3	5	5	5	5	5	5	5	7	7	7
15	BLY															1	3	3	5	5	5	5	5	5	5	7	7	7
16	BLZ																1	3	5	5	5	5	5	5	5	5	7	7
17	BMY																	1	5	5	5	5	5	5	5	5	7	7
18	BMZ																		1	5	5	5	5	5	5	5	5	5
19	CKX																			1	3	5	5	5	5	5	5	5
20	CLX																				1	5	5	5	5	5	5	5
21	CKY																					1	3	3	5	5	5	5
22	CKZ																						1	3	5	5	5	5
23	CMX																							1	5	5	5	5
24	CLY																								1	3	3	3
25	CLZ																									1	3	3
26	CMY																										1	3
27	CMZ																											1

|	
3,5,7,9	changes induced by re-deviding categories
3,5	changes corresponding to the pairwise comparison of neighboring categories
5	changes made to maintain consistency after the previous group of changes

The Saaty's Method

2 ways for calculation scores of categories from the given matrix S:

- **Eigenvector Method** - if S is close enough to an ideally consistent matrix, the scores of categories are calculated as components of the eigenvector corresponding to the largest eigenvalue of the matrix S.

- **Geometric Means Method**
  - The columns of S can be interpreted as repeated measurements of the relative importance among the 27 artistic categories (**compositional data**).
  - A proper estimator of the expected value of this kind of data is geometric mean (geometric means of rows of S are calculated).
  - The weaker consistency of S represents a **natural requirement given on expertly defined data**.

The Saaty's Method – determined scores

Category	Relevance or significance	Extent	Institutional reception	Eigenvector method	Geom. means method
AKX	crucial significance	large	international	305	305
AKY	crucial significance	large	national	259	254
AKZ	crucial significance	large	regional	210	217
ALX	crucial significance	medium	international	191	194
AMX	crucial significance	limited	international	174	171
ALY	crucial significance	medium	national	138	138
ALZ	crucial significance	medium	regional	127	124
BKX	containing numerous important innovations	large	international	117	112
AMY	crucial significance	limited	national	97	94
AMZ	crucial significance	limited	regional	90	87
BKY	containing numerous important innovations	large	national	79	75
BKZ	containing numerous important innovations	large	regional	66	66
BLX	containing numerous important innovations	medium	international	62	61
BMX	containing numerous important innovations	limited	international	48	50
BLY	containing numerous important innovations	medium	national	44	46
BLZ	containing numerous important innovations	medium	regional	40	41
BMY	containing numerous important innovations	limited	national	37	38
BMZ	containing numerous important innovations	limited	regional	31	30
CKX	pushing forward modern trends	large	international	26	26
CLX	pushing forward modern trends	medium	international	24	24
CKY	pushing forward modern trends	large	national	19	20
CKZ	pushing forward modern trends	large	regional	17	18
CMX	pushing forward modern trends	limited	international	16	16
CLY	pushing forward modern trends	medium	national	12	13
CLZ	pushing forward modern trends	medium	regional	10	11
CMY	pushing forward modern trends	limited	national	9	9
CMZ	pushing forward modern trends	limited	regional	8	9

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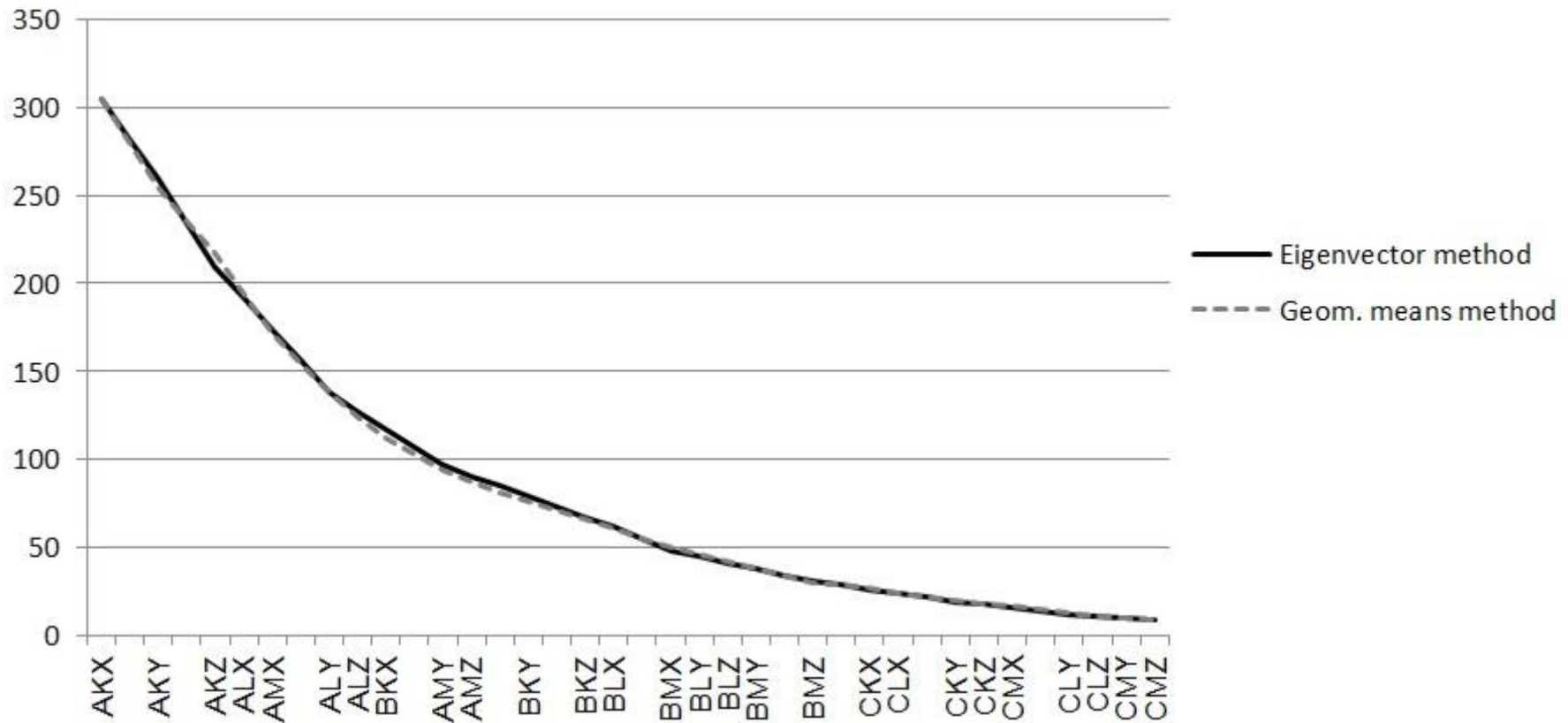
The Saaty's Method

Conclusion

- RAP and the corresponding methodology are **currently being pilot-tested in the Czech Republic**

- Intention to find the **best possible conversion of preferences of experts into scores** (in all segments of art).

- Saaty's method used in a **special procedure**

  - **Criteria dependency**

  - **Expert opinion partially inconsistent**

- Work continues

  - **refining the triplets of class specification** for all three criteria and for all the fields of art, **mutual comparability**

  - developing a most **objective mechanism of expert classification** of artistic production into 27 categories.

Thanks for your attention!

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