Transformed Weibull Distribution in Survival Analysis

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SURVIVAL ANALYSIS

= study of data from a well-defined time origin until the occurrence of some particular event or end-point

- Survival time T (t)

- Censoring

  = the end-point of the interest has not been observed

SURVIVAL ANALYSIS

- Survival function

  = probability, that an individual survives longer than t

$$S(t) = P(T > t) = 1 - F(t)$$

- Hazard function

  = probability that an individual dies at time t, conditional on he or she having survived to that time

$$h(t) = \frac{f(t)}{S(t)} = -\frac{d}{dt}[log S(t)]$$

METHODS

- Non-parametric

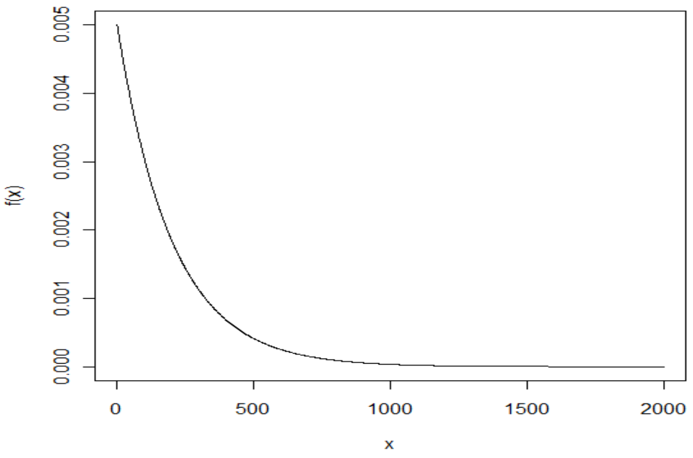
  - Kaplan-Meier estimate

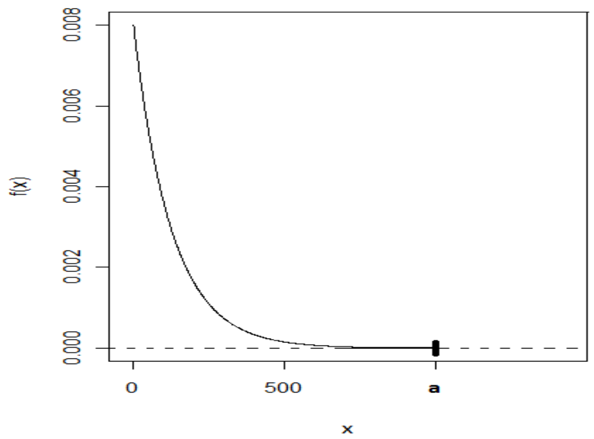
$$\hat{S}(t) = \prod_{j=1}^{k} \frac{n_j - d_j}{n_j}$$

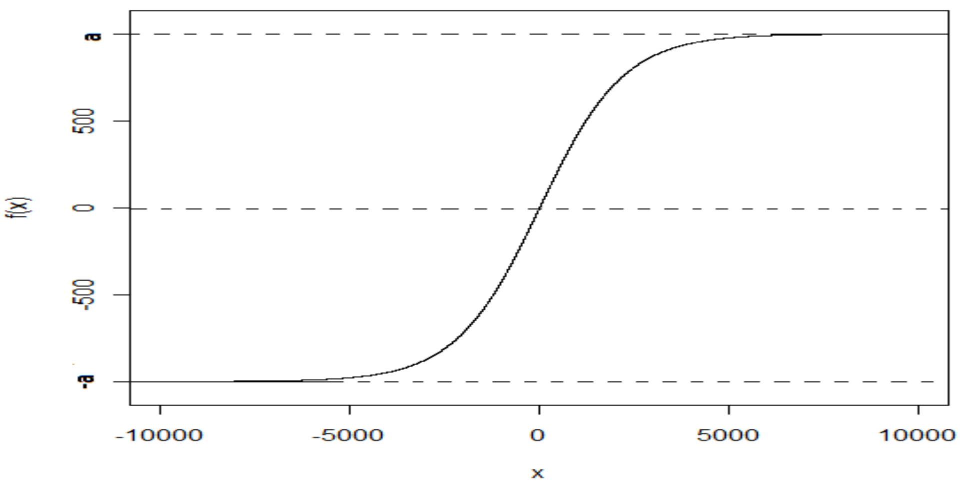
- Parametric

  - Exponential, Weibull, Log-normal, Gamma distributions

TRANSFORMATION







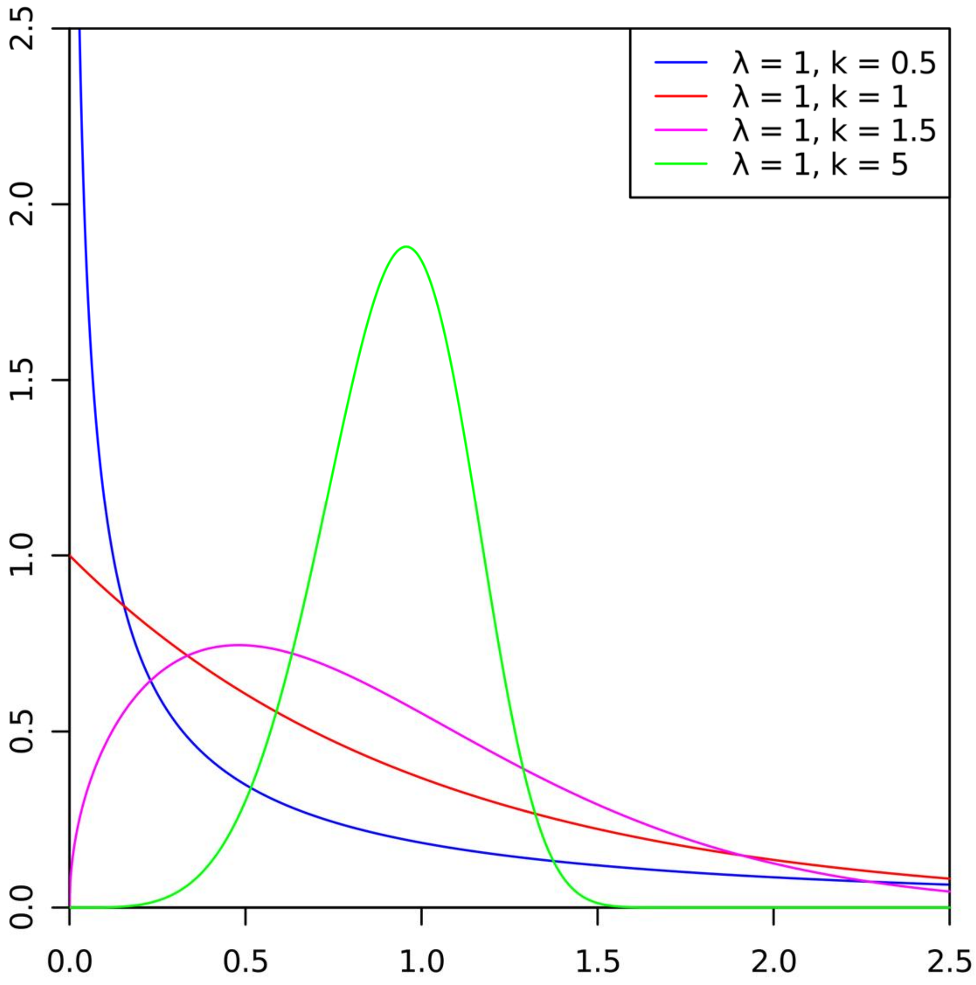
WEIBULL DISTRIBUTION

$$f(t) = \frac{b}{\lambda^b} t^{b-1} e^{-\left(\frac{t}{\lambda}\right)^b}$$

$$F(t) = 1 - e^{-\left(\frac{t}{\lambda}\right)^b}$$

$$S(t) = 1 - F(t) = e^{-\left(\frac{t}{\lambda}\right)^b}$$

$$h(t) = \frac{f(t)}{S(t)} = \frac{b}{\lambda^b} t^{b-1}$$

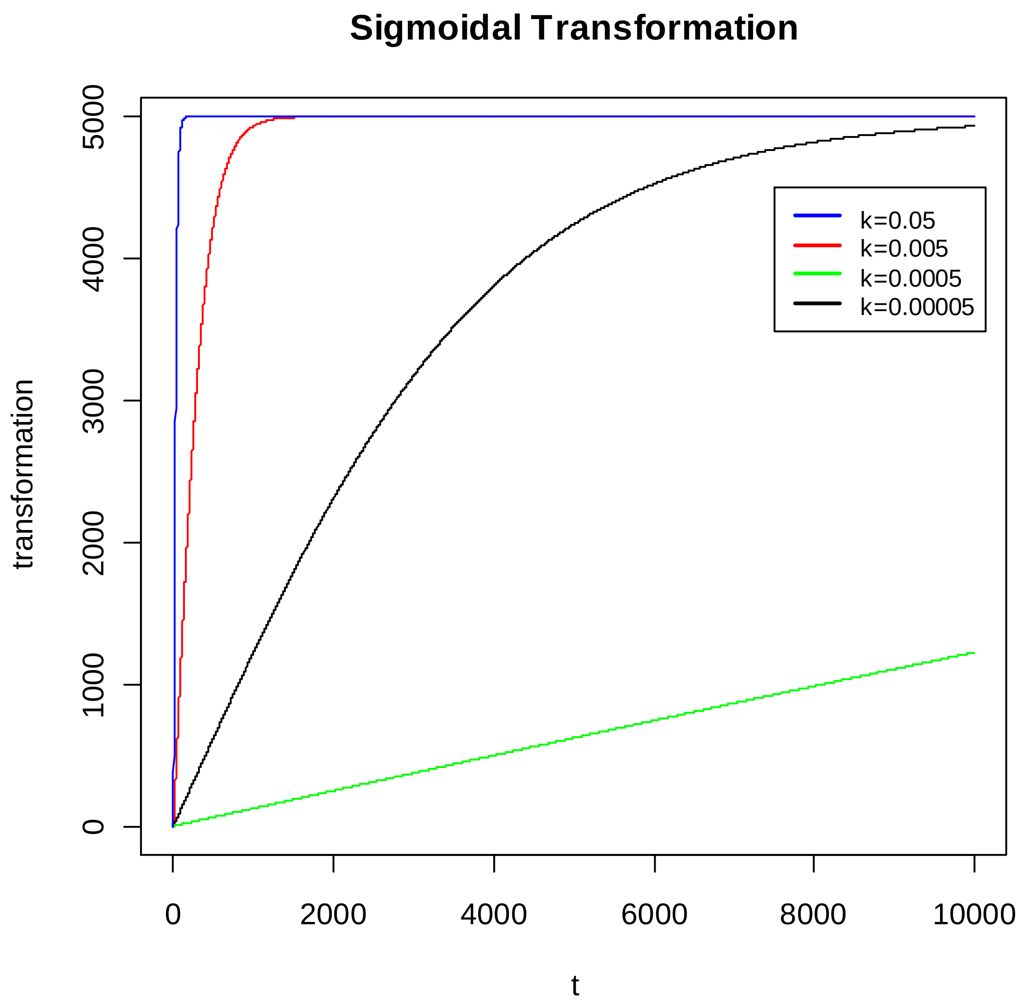


SIGMOIDAL TRANSFORMATION

$$y = \frac{a}{1 + e^{-kt}}$$

$$y = \frac{2a}{1 + e^{-kt}} - a$$

$$y = -\frac{1}{k} ln \frac{a - t}{a + t}$$



TRANSFORMED WEIBULL DISTRIBUTION

$$f(t) = \frac{2ab}{l(a^2 - t^2)} \cdot \left(-\frac{1}{l} ln \frac{a-t}{a+t}\right)^{b-1} \cdot e^{-\left(-\frac{1}{l} ln \frac{a-t}{a+t}\right)^{b}}$$

$$F(t) = 1 - e^{-\left(-\frac{1}{l} ln \frac{a-t}{a+t}\right)^{b}}$$

$$S(t) = 1 - F(t) = e^{-\left(-\frac{1}{l} ln \frac{a-t}{a+t}\right)^{b}}$$

$$h(t) = \frac{f(t)}{S(t)} = \frac{2ab}{l(a^2 - t^2)} \cdot \left(-\frac{1}{l} ln \frac{a-t}{a+t}\right)^{b-1}$$

$$l = k\lambda$$

MAXIMUM LIKELIHOOD METHOD

$$l\big((\boldsymbol{t},\boldsymbol{c}),\boldsymbol{\beta}\big) = \prod_{i=1}^{n} \{[f(t_i,\boldsymbol{\beta})]^{c_i}[S(t_i,\boldsymbol{\beta})]^{1-c_i}\}$$

$$L\big((\boldsymbol{t},\boldsymbol{c}),\boldsymbol{\beta}\big) = \sum_{i=1}^{n} \{c_i ln[f(t_i,\boldsymbol{\beta})] + (1-c_i) ln[S(t_i,\boldsymbol{\beta})]\}$$

$$L = \sum_{i=1}^{n} \left\{ c_i ln\left(\frac{2ab}{l(a^2-t_i^2)} \cdot \left(-\frac{1}{l} ln\frac{a-t_i}{a+t_i}\right)^{b-1} \cdot e^{-\left(-\frac{1}{l} ln\frac{a-t_i}{a+t_i}\right)^{b}} \right) + (1-c_i) ln\left(e^{-\left(-\frac{1}{l} ln\frac{a-t_i}{a+t_i}\right)^{b}} \right) \right\}$$

MAXIMUM LIKELIHOOD METHOD

$$\sum_{i=1}^{n} \frac{c_i - b\ln\left(-\frac{ln\frac{a-t_i}{a+t_i}}{l}\right)\left(\left(-\frac{ln\frac{a-t_i}{a+t_i}}{l}\right)^b - c_i\right)}{b} = 0$$

$$\sum_{i=1}^{n} \frac{b\left(\left(-\frac{ln\frac{a-t_i}{a+t_i}}{l}\right)^b - c_i\right)}{l} = 0$$

$$-\sum_{i=1}^{n} \frac{c_i(a^2+t_i^2)ln\frac{a-t_i}{a+t_i} + 2at_i\left(b\left(-\frac{ln\frac{a-t_i}{a+t_i}}{l}\right)^b - bc_i + c_i\right)}{a(a^2-t_i^2)ln\frac{a-t_i}{a+t_i}} = 0$$

ESTIMATIONS

Used data - 333 women with the breast cancer in the fourth stadium

- 8 censored observations

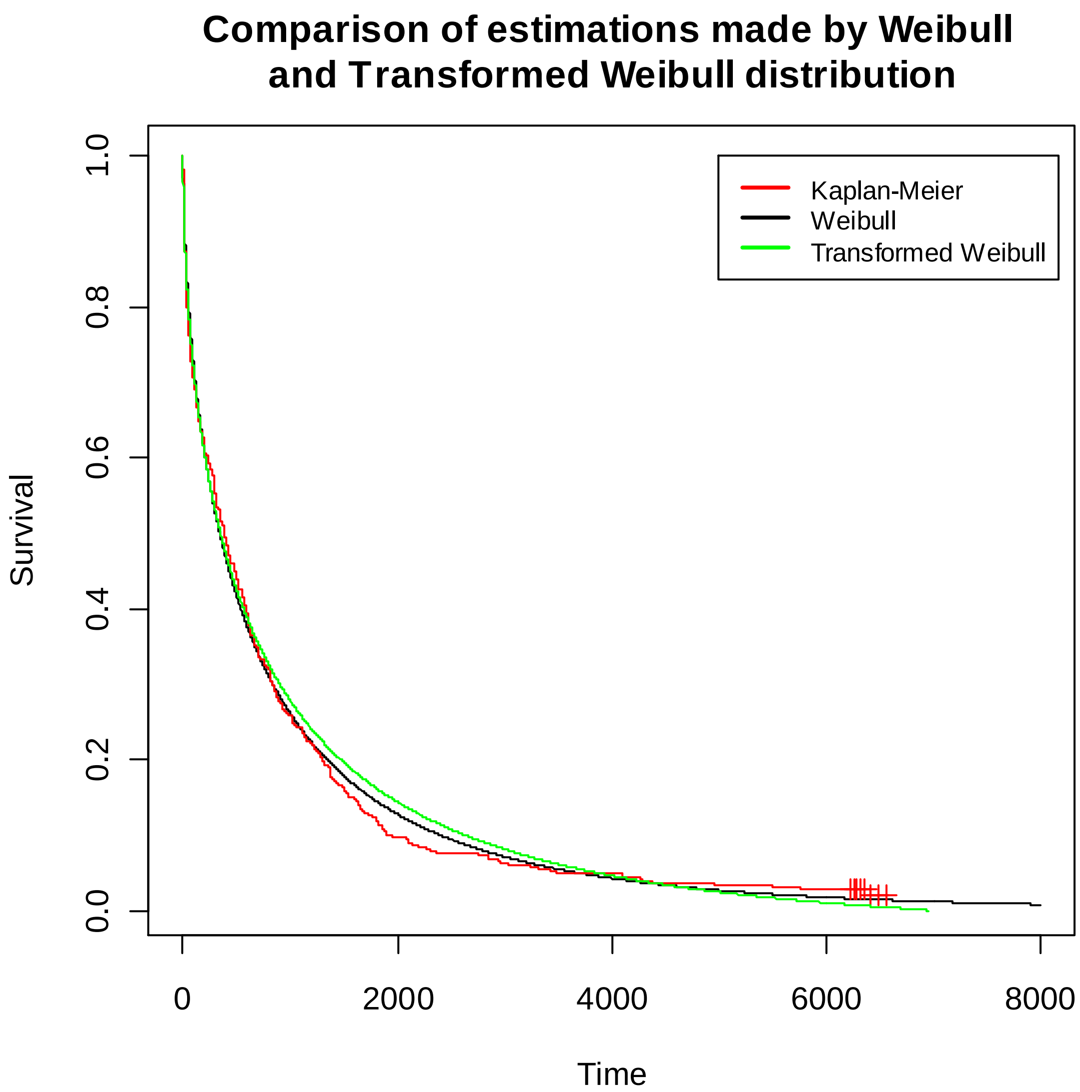
Estimations of the parameters:

$$\hat{b} = 0{,}5815282$$

$$\hat{l} = 0{,}1894694$$

$$\hat{a} = 6951{,}7905143$$

SURVIVAL FUNCTION



NEXT STEPS

- ☑ Other optimalization function
- ☑ More suitable distribution
- ☑ New transformation function

Thank you for your attention