The mathematical model of maximum outflow of gas depending on the length of blow-off pipe

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– Calculation of critical pressure and maximal outflow for specific pipe length
– Comparison of critical condition for real and ideal gas

Motivation

WHERE?

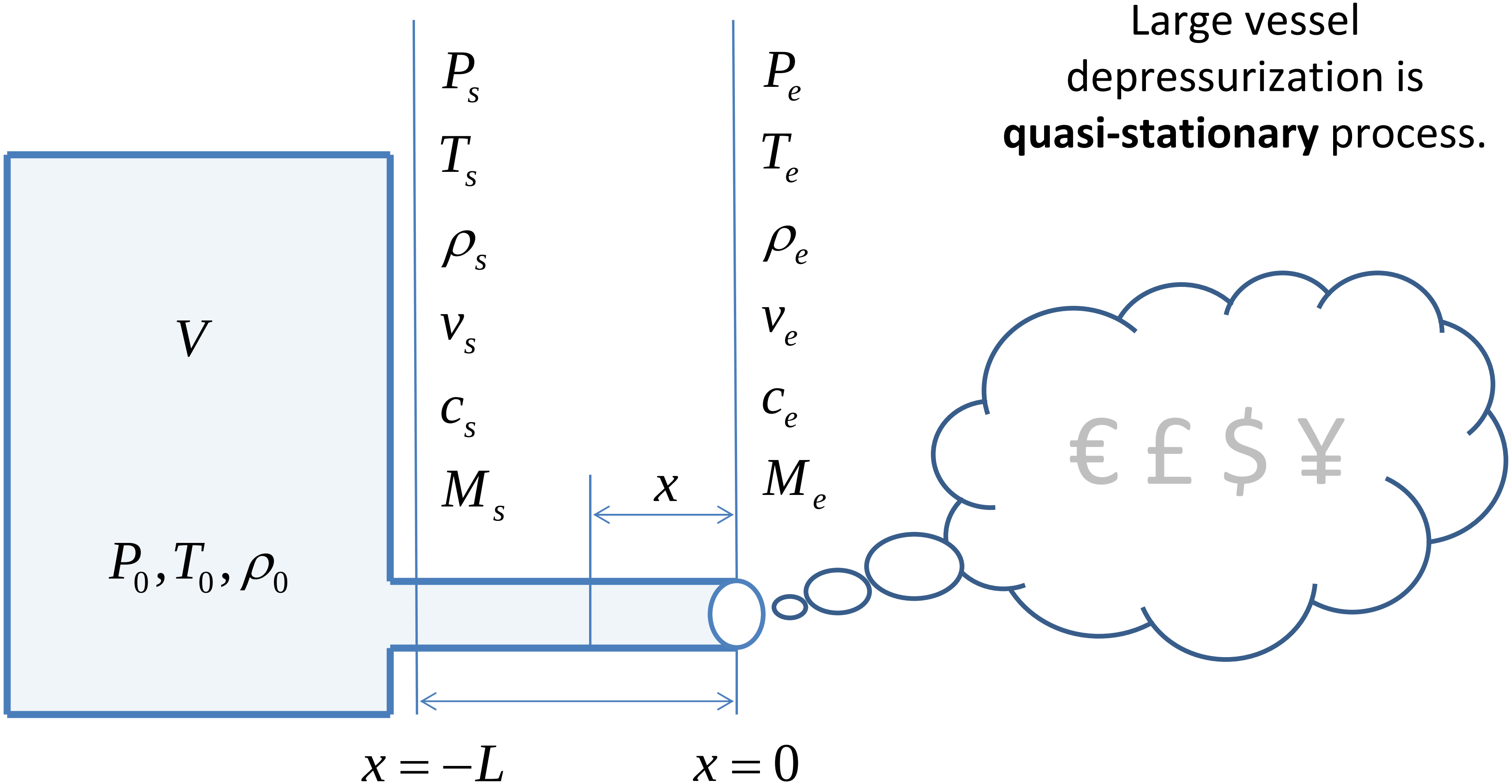
- Chemical industry
- Industrial complexes and pipeline systems
- Gas transport

WHY?

- Blow-through of compressor pipeline yard
- Emergency and repair depressurization

Depressurization Scheme

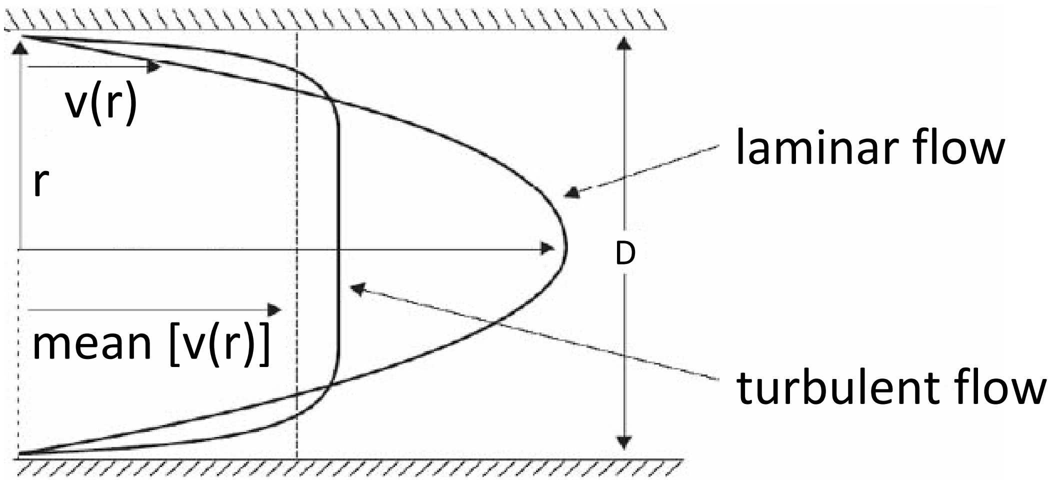
Gas properties: R, Z, μ, c_P, κ



Large vessel depressurization is **quasi-stationary** process.

Model of Blow-off Pipe

Re > 2300 => turbulent flow



$$M = \frac{v}{c}$$

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\sigma}{3{,}710D} + \frac{4{,}518}{\mathrm{Re}}\log\frac{\mathrm{Re}}{7}\right)$$

Mathematical Model of Gas Flow in Pipe

Complete system of conservation laws for one-dimension stationary model of gas flow

MASS:

$$\frac{1}{\rho}\frac{\partial \rho}{\partial x} + \frac{1}{v}\frac{\partial v}{\partial x} + \frac{1}{S}\frac{\partial S}{\partial x} = 0$$

MOMENTUM:

$$v\frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \lambda\frac{v^2}{2D_H}$$

ENERGY:

$$v\frac{\partial v}{\partial x} + \frac{\partial h}{\partial x} = 0$$

ENTROPY vs. ENTHALPY:

$$\frac{\partial h}{\partial x} = c_P\frac{\partial T}{\partial x} = \frac{1}{\rho}\frac{\partial P}{\partial x} + T\frac{\partial s}{\partial x}$$

Mathematical Model of Gas Flow Depending on Pipe Length

Transformation of temperature to kinetic energy

Resistance caused by roughness of inner pipe wall

Model for very short pipe	Model for short pipe	Model for very long pipe
$L/D \ll 100$	$L/D < 1000$	$L/D > 1000$
conservation of **MASS** conservation of **ENERGY**	conservation of **MASS** conservation **MOMENTUM** conservation of **ENERGY**	conservation of **MASS** conservation **MOMENTUM**
Isentropic process	Non-Isentropic process	Non-Isentropic process

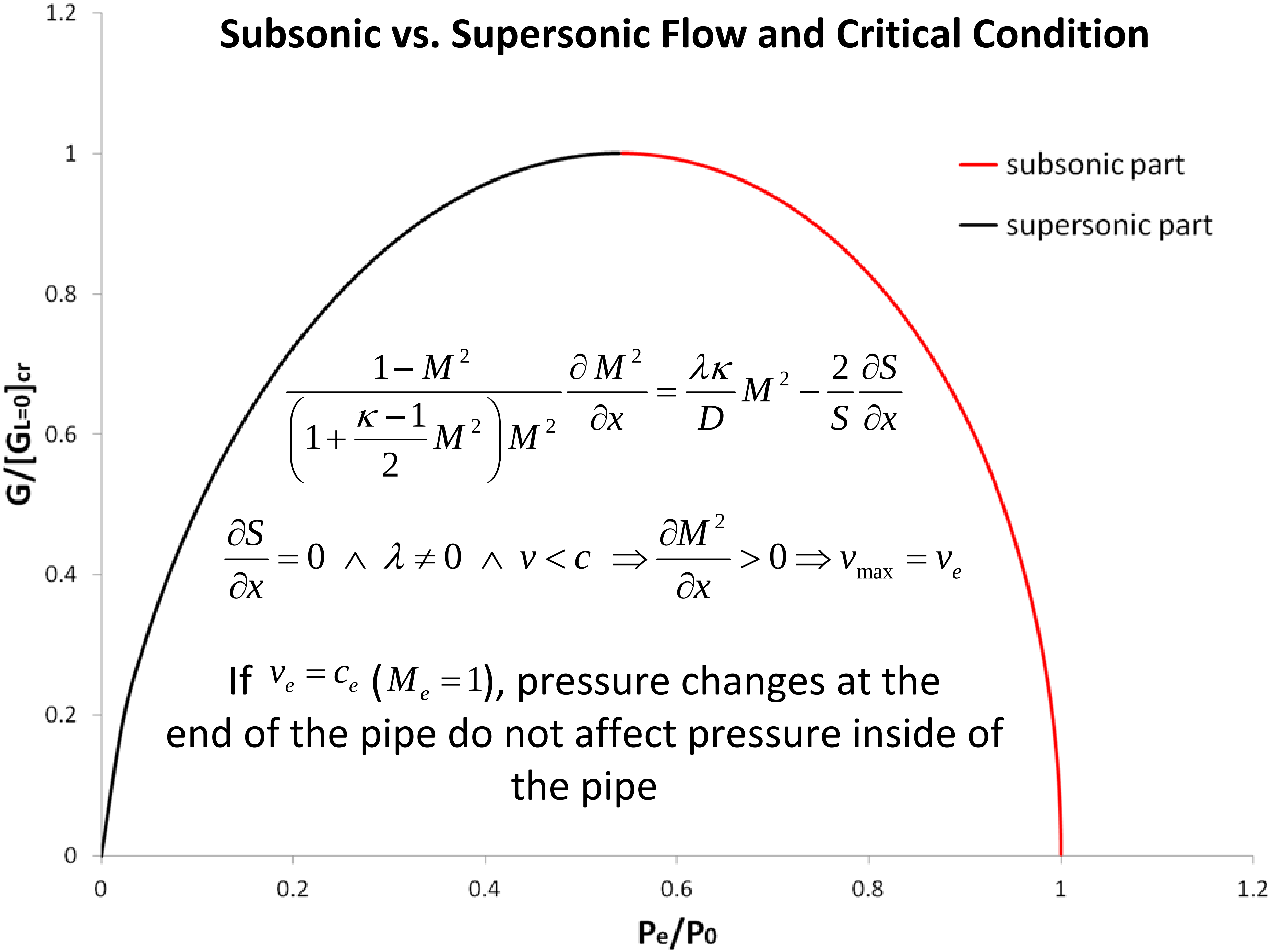
Oswatitsch Equations

Complete system of conservation laws for one-dimensional stationary gas flow leads to Oswatitsch equations

$$\frac{\kappa\lambda}{D}L = \frac{1}{M_s^{\,2}} - \frac{1}{M_e^{\,2}} + \frac{\kappa+1}{2}\ln\left[\frac{1-\frac{2}{\kappa+1}\left(1-\frac{1}{M_e^{\,2}}\right)}{1-\frac{2}{\kappa+1}\left(1-\frac{1}{M_s^{\,2}}\right)}\right]$$

$$G = M\rho S\sqrt{\kappa RT} = MPS\sqrt{\frac{\kappa}{RT}} = MPS\sqrt{\frac{\kappa}{RT_0}\left(1+\frac{\kappa-1}{2}M^2\right)}$$

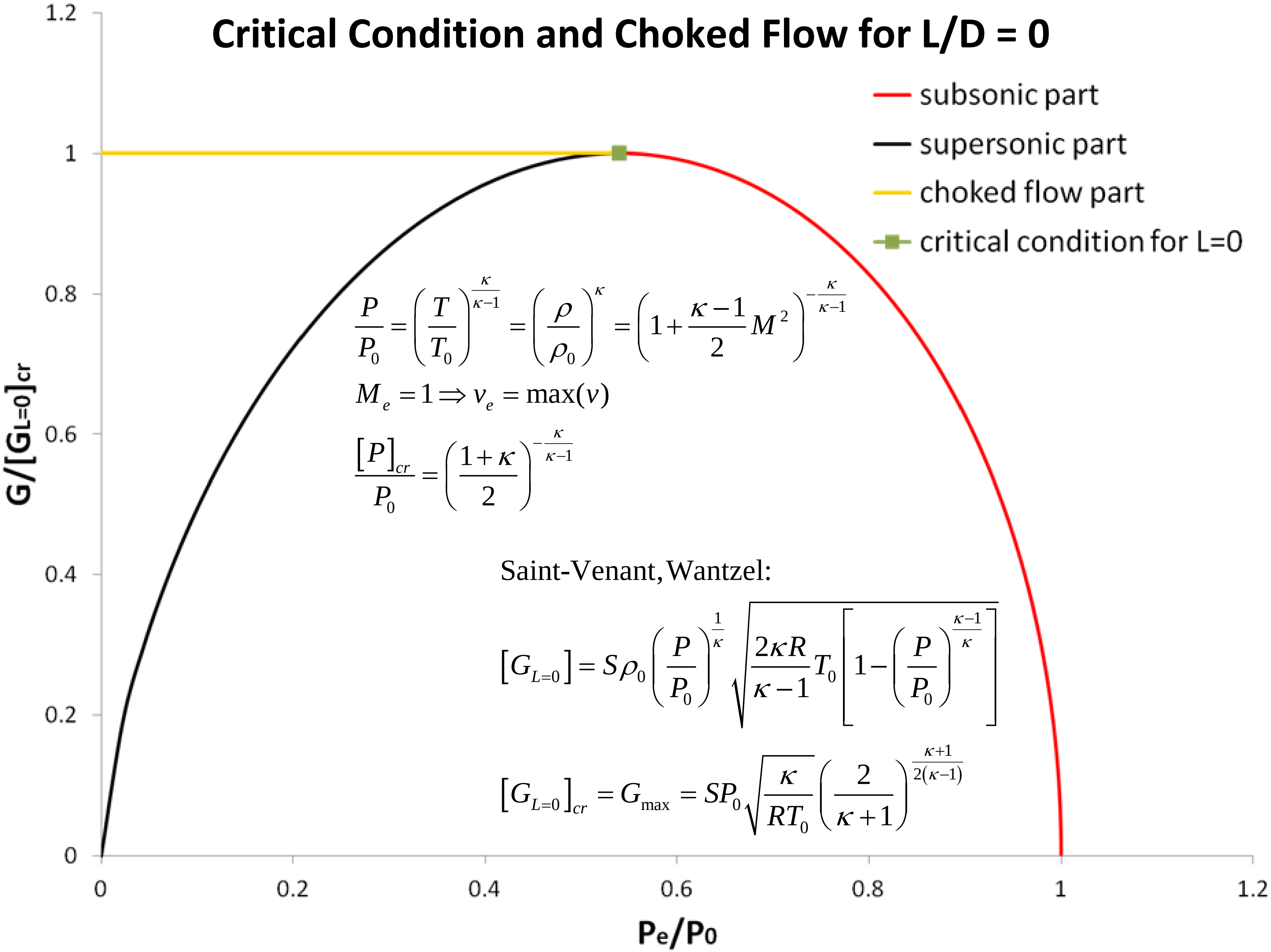
Subsonic vs. Supersonic Flow and Critical Condition

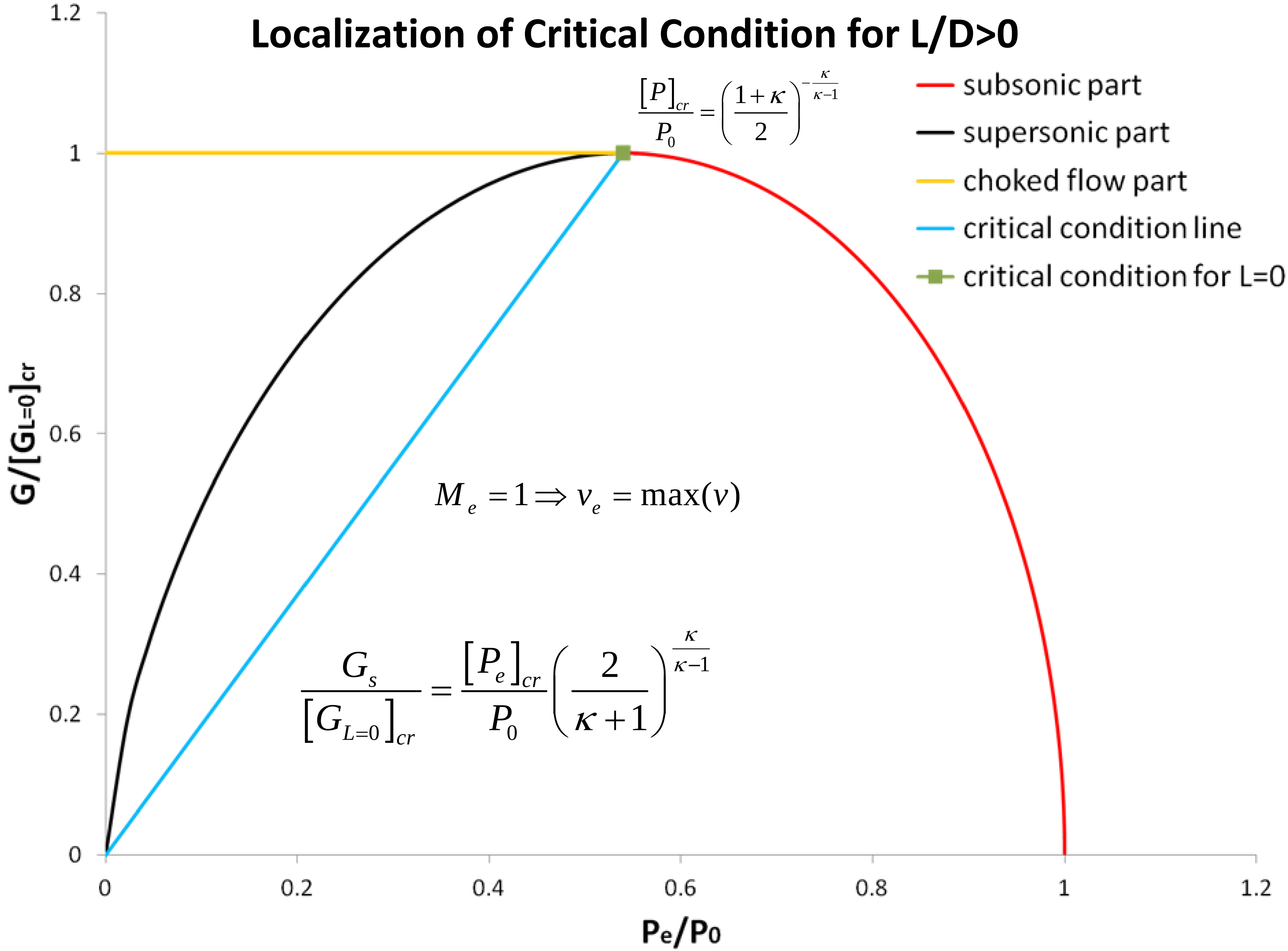


$$\frac{1-M^2}{\left(1+\frac{\kappa-1}{2}M^2\right)M^2}\frac{\partial M^2}{\partial x}=\frac{\lambda\kappa}{D}M^2-\frac{2}{S}\frac{\partial S}{\partial x}$$

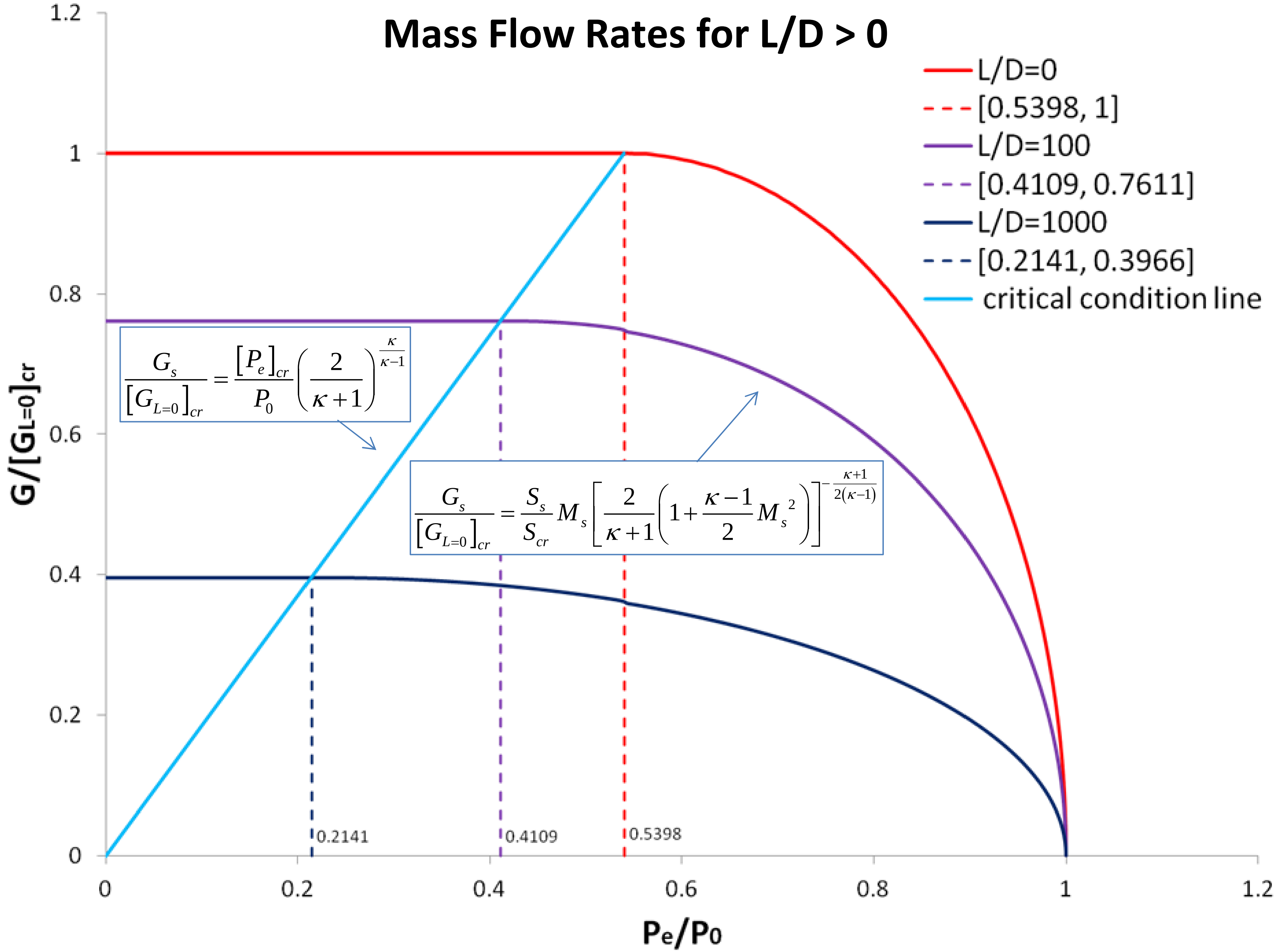
$$\frac{\partial S}{\partial x}=0 \ \wedge\ \lambda\neq 0 \ \wedge\ v<c \ \Rightarrow \frac{\partial M^2}{\partial x}>0 \Rightarrow v_{\max}=v_e$$

If $v_e = c_e$ ($M_e = 1$), pressure changes at the end of the pipe do not affect pressure inside of the pipe


Critical Condition and Choked Flow for L/D = 0
subsonic part
supersonic part
choked flow part
critical condition for L=0
$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{\kappa}{\kappa-1}} = \left(\frac{\rho}{\rho_0}\right)^{\kappa} = \left(1+\frac{\kappa-1}{2}M^2\right)^{-\frac{\kappa}{\kappa-1}}$
$M_e = 1 \Rightarrow v_e = \max(v)$
$\frac{[P]_{cr}}{P_0} = \left(\frac{1+\kappa}{2}\right)^{-\frac{\kappa}{\kappa-1}}$
Saint-Venant, Wantzel:
$[G_{L=0}] = S\rho_0 \left(\frac{P}{P_0}\right)^{\frac{1}{\kappa}} \sqrt{\frac{2\kappa R}{\kappa-1} T_0 \left[1-\left(\frac{P}{P_0}\right)^{\frac{\kappa-1}{\kappa}}\right]}$
$[G_{L=0}]_{cr} = G_{\max} = SP_0 \sqrt{\frac{\kappa}{RT_0}} \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{2(\kappa-1)}}$
$G/[G_{L=0}]_{cr}$
P_e/P_0
0
0.2
0.4
0.6
0.8
1
1.2



Localization of Critical Condition for L/D>0
$\frac{[P]_{cr}}{P_0} = \left(\frac{1+\kappa}{2}\right)^{-\frac{\kappa}{\kappa-1}}$
subsonic part
supersonic part
choked flow part
critical condition line
critical condition for L=0
$M_e = 1 \Rightarrow v_e = \max(v)$
$\frac{G_s}{[G_{L=0}]_{cr}} = \frac{[P_e]_{cr}}{P_0}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$
G/[GL=0]cr
Pe/P0
0
0.2
0.4
0.6
0.8
1
1.2



Mass Flow Rates for L/D > 0
L/D=0
[0.5398, 1]
L/D=100
[0.4109, 0.7611]
L/D=1000
[0.2141, 0.3966]
critical condition line
$\frac{G_s}{[G_{L=0}]_{cr}} = \frac{[P_e]_{cr}}{P_0}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}$
$\frac{G_s}{[G_{L=0}]_{cr}} = \frac{S_s}{S_{cr}} M_s \left[\frac{2}{\kappa+1}\left(1+\frac{\kappa-1}{2}M_s^{\ 2}\right)\right]^{-\frac{\kappa+1}{2(\kappa-1)}}$
0.2141
0.4109
0.5398
$G/[G_{L=0}]_{cr}$
P_e/P_0
0
0.2
0.4
0.6
0.8
1
1.2

Results: Equations

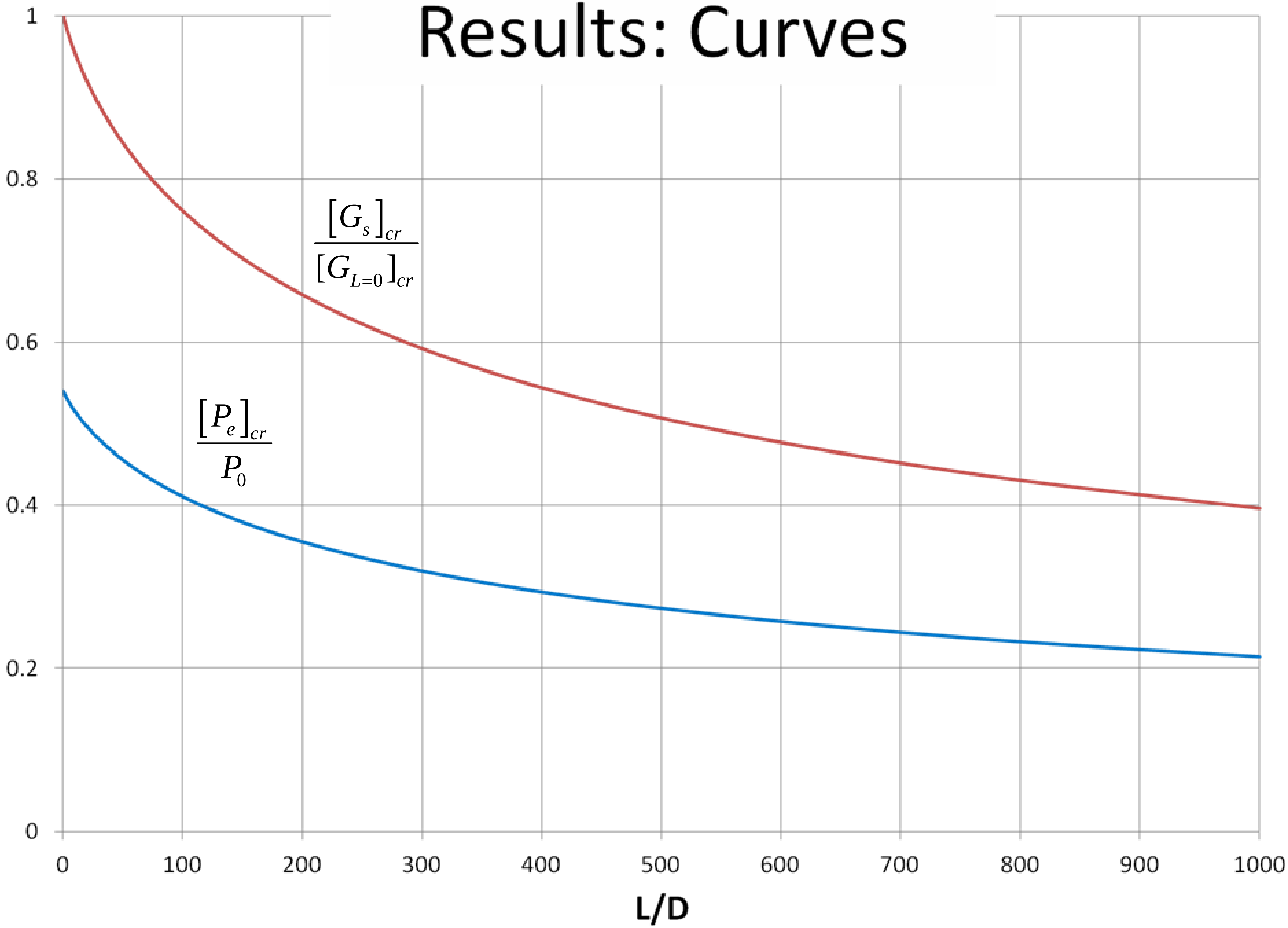
$$M_s = Root\left(1-\frac{1}{{M_s}^2}+\frac{\kappa+1}{2}\ln\left[1-\frac{2}{\kappa+1}\left(1-\frac{1}{{M_s}^2}\right)\right]+\frac{\kappa\lambda}{D}L\right)=\sqrt{\frac{\frac{2}{\kappa+1}}{\frac{2}{\kappa+1}-W\left(\frac{-1}{e^{\frac{2\kappa\lambda L}{D(\kappa+1)}+1}}\right)-1}}$$

$$\frac{[P_e]_{cr}}{P_0}=M_s\left[\frac{2}{\kappa+1}\left(1+\frac{\kappa-1}{2}{M_s}^2\right)\right]^{-\frac{\kappa+1}{2(\kappa-1)}}\left(\frac{2}{\kappa+1}\right)^{-\frac{\kappa}{\kappa-1}}$$

$$[G_s]_{cr}=SP_0\sqrt{\frac{\kappa}{RT_0}}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{2(\kappa-1)}}M_s\left[\frac{2}{\kappa+1}\left(1+\frac{\kappa-1}{2}{M_s}^2\right)\right]^{-\frac{\kappa+1}{2(\kappa-1)}}$$

where $W(z)$ is negative real part of Lambert W function

Results: Curves



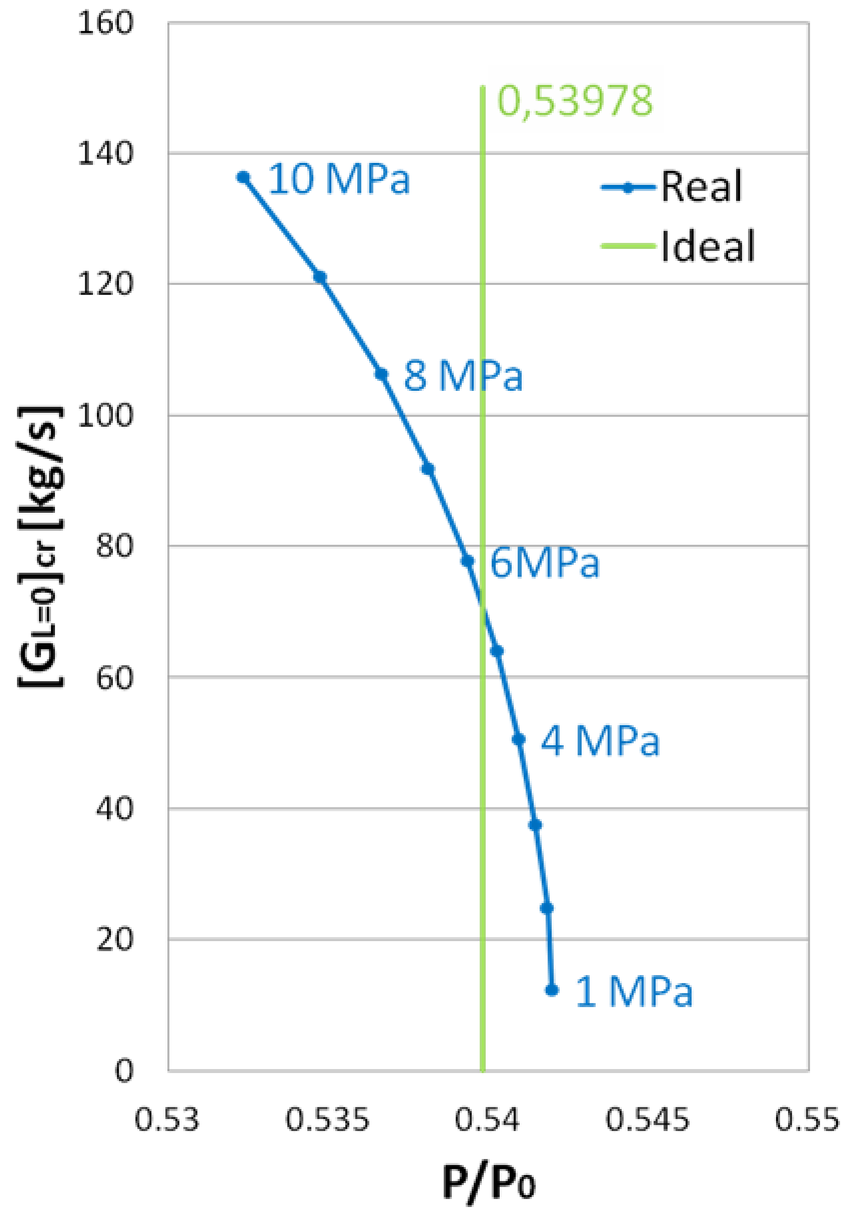
Ideal vs. Real Gas (D = 0,1 m, T_0 = 20 °C)

AGA 10: $s(P,T),\ H(P,T),\ Z(P,T)$

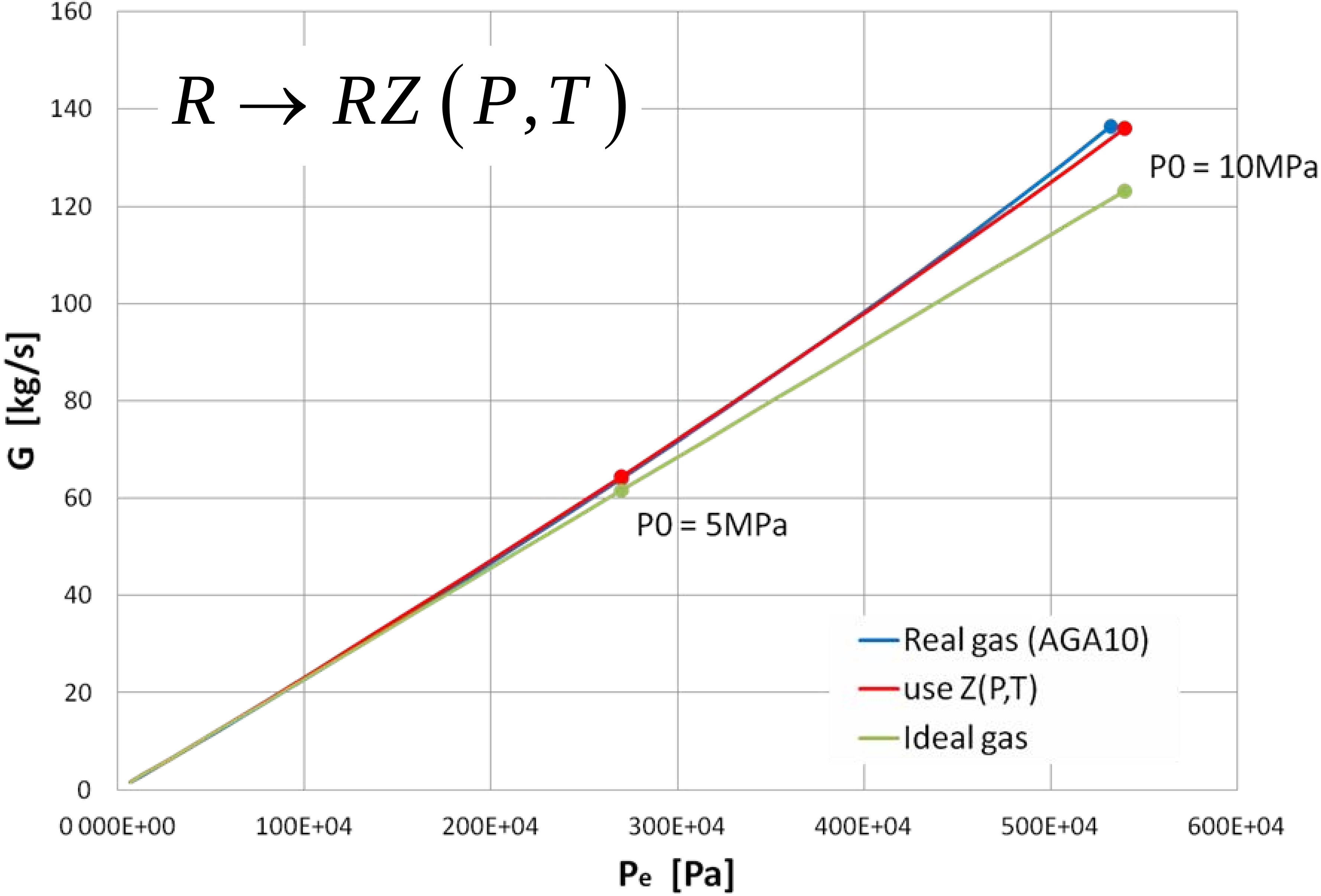
$$s(P_0,T_0) = s(P,T),$$

where $P_0,\ T_0$ are known. We use entropy equality to find T for chosen P/P_0.

$$G = S\rho v = S\frac{P\sqrt{2\left[H(P_0,T_0) - H(P,T)\right]}}{Z(P,T)RT}$$



Ideal vs. Real Gas (D = 0,1 m, T_0 = 20 °C)



Thank you for your attention.