IT4Innovations
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Structure of generalized intermediate syllogisms

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INVESTICE DO VAŠÍ BUDOUCNOSTI

OP Výzkum a vývoj pro inovace

Outline

Motivation and main goals

Motivation for this research

- Elaboration of theory of intermediate quantifiers from Peterson's book Intermediate Quantifiers - where Peterson analyzed the main intermediate quantifiers.
- In the book of Peterson is no formal mathematical system.
- Application of Łukasewicz fuzzy type theory.
- The first goal is to define new intermediate quantifier "more then half" and also to prove 19 new intermediate generalized syllogisms with this quantifier.
- The second goal is to find the main strongly valid syllogisms for every Figure and to show that all the 144 intermediate generalized syllogisms are strongly valid in our theory.

Structure of truth values-MV$_\Delta$-algebra

MV$_\Delta$-algebra

$$\mathcal{L}. = \langle L, \vee, \wedge, \otimes, \rightarrow, \mathbf{0}, \mathbf{1}, \Delta \rangle, \tag{1}$$

1. $\langle L, \vee, \wedge, \otimes, \rightarrow, \mathbf{0}, \mathbf{1}, \rangle$ is an MV-algebra with involutive negation,

where

- $\Delta a \vee \neg \Delta a = 1$,
- $\Delta(a \vee b) \leq \Delta a \vee \Delta b$,
- $\Delta a \leq a, \qquad \Delta a \leq \Delta\Delta a$,
- $\Delta(a \rightarrow b) \leq \Delta a \rightarrow \Delta b$,
- $\Delta \mathbf{1} = \mathbf{1}$.

Example of MV$_\Delta$-algebra

Standard Łukasiewicz algebra

$$\mathcal{L} = \langle [0,1], \vee, \wedge, \otimes, \rightarrow, 0, 1, \Delta \rangle \tag{2}$$

1. $\vee = \max$
2. $\wedge = \min$
3. $a \otimes b = \max(0, a + b - 1)$
4. $a \rightarrow b = 1 \wedge (1 - a + b)$
5. $\neg a = a \rightarrow 0 = 1 - a$

Basic syntactical elements

The language of Ł-FTTdenoted by J consists of:

- variables $x_\alpha, \ldots$
- special constants $c_\alpha, \ldots$ ($\alpha \in$ *Types*)
- λ and brackets
- $\mathbf{E}_{(o\alpha)\alpha}$ for every $\alpha \in$ *Types* for fuzzy equality,
- $\mathbf{C}_{(oo)o}$ for conjunction,
- $\mathbf{D}_{(oo)}$ for delta operation.

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Basic definitions

1. Equivalence: $\equiv \; := \lambda x_\alpha \lambda y_\alpha (\mathbf{E}_{(o\alpha)\alpha} y_\alpha) x_\alpha, \qquad \alpha \in$ *Types*.
2. Conjunction: $\wedge \; := \lambda x_o \lambda y_o (\mathbf{C}_{(oo)o} y_o) x_o$.
3. Delta connective: $\boldsymbol{\Delta} \; := \lambda x_o \mathbf{D}_{oo} x_o$.

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Derived connectives

1. Representation of truth: $\top := \lambda x_o x_o \equiv \lambda x_o x_o$.
2. Representation of falsity: $\bot := \lambda x_o x_o \equiv \lambda x_o \top$.
3. Negation: $\neg := \lambda x_o (x_o \equiv \bot)$.
4. Implication: $\Rightarrow \;:= \lambda x_o \lambda y_o \,(x_o \wedge y_o) \equiv x_o$
5. $\&, \nabla, \vee$ are defined as in Łukasiewicz logic.
6. General quantifier: $(\forall x_\alpha) A_o := (\lambda x_\alpha A_o \equiv \lambda x_\alpha \top)$,
7. Existential quantifier: $(\exists x_\alpha) A_o := \neg(\forall x_\alpha) \neg A_o$.

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Axioms and inference rules in Ł-FTT

- 17 axioms
- two inference rules where the rules *modus ponens and generalization* are the rules derivative.

Semantics in Ł-FTT

- A *frame* is a tuple

$$\mathcal{M} = \langle (M_\alpha, =_\alpha)_{\alpha \in \mathit{Types}}, \mathcal{L}_\Delta \rangle$$

1. $(M_\alpha)_{\alpha \in \mathit{Types}}$ is a basic frame
2. $\mathcal{L}_\Delta$ is MV-algebra with $\boldsymbol{\Delta}$
3. $=_\alpha$ is a fuzzy equality on M_α.

- We say that a frame $\mathcal{M}$ is a *model* of a theory T if all axioms are true in the degree $\mathbf{1}$ in $\mathcal{M}$.

Trichotomous evaluative linguistic expressions

TEE

- are special expressions of natural language, e.g., *small, big, about fourteen, very short, more or less deep, not thick*.
- Linguistic hedge can be
  - *narrowing — extremely, significantly, very*
  - *widening — more or less, roughly, quite roughly, very roughly*
  - *empty hedge*
- We will work with expressions: extremely big, very big, very roughly, not small.
- T^{Ev} has 11 axioms.

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- T^{Ev} has 11 axioms.

Theory of intermediate quantifiers T^{IQ}

1. is a special theory of Ł-FTT extending the theory T^{Ev} of evaluative linguistic expressions
2. we consider a special formula μ of type $o(o\alpha)(o\alpha)$ such that values of the measure are taken from the set of truth values
3. μ has four axioms

Definition of intermediate generalized quantifiers

Definitions of intermediate generalized quantifiers of the form "Quantifier B's are A"

(a) $(Q^{\forall}_{Ev}\, x)(B,A) := (\exists z)((\Delta(z \subseteq B)\, \&(\forall x)(z\, x \Rightarrow Ax)) \wedge Ev((\mu B)z)),$

(b) $(Q^{\exists}_{Ev}\, x)(B,A) := (\exists z)((\Delta(z \subseteq B)\, \&(\exists x)(zx \wedge Ax)) \wedge Ev((\mu B)z)).$

Definition of intermediate generalized quantifiers

Explanation of definition of IGQ

Each formula above consists of three parts:

$$\underbrace{(\exists z)((\boldsymbol{\Delta}(z \subseteq B)}_{\text{“the greatest” part of } B\text{'s}} \quad \& \quad \underbrace{(\forall x)(z\,x \Rightarrow Ax))}_{\text{each } z\text{'s has } A} \quad \wedge \quad \underbrace{Ev((\mu B)z))}_{\text{size of } z \text{ is evaluated by } Ev} \tag{3}$$

Definition of intermediate generalized quantifiers with presupposition

Interpretation of "Quantifier B's are A" with presupposition

(a) $({}^{*}Q^{\forall}_{Ev}\, x)(B,A) \equiv (\exists z)((\boldsymbol{\Delta}(z \subseteq B)\, \&\, (\exists x)zx\, \&\, (\forall x)(z\, x \Rightarrow Ax)) \wedge Ev((\mu B)z))$,

(b) $({}^{*}Q^{\exists}_{Ev}\, x)(B,A) := (\exists z)((\boldsymbol{\Delta}(z \subseteq B)\, \&\, (\exists x)zx\, \&\, (\exists x)(zx \wedge Ax)) \wedge Ev((\mu B)z))$.

where only non-empty subsets of B are considered.

“All”, “No”, “Almost all”, “Few”, “Most”

A: All B are $A := Q^{\forall}_{Bi\Delta}(B, A) \equiv (\forall x)(Bx \Rightarrow Ax)$,

E: No B are $A := Q^{\forall}_{Bi\Delta}(B, \neg A) \equiv (\forall x)(Bx \Rightarrow \neg Ax)$,

P: Almost all B are $A := Q^{\forall}_{Bi\,Ex}(B, A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) \,\&\, (\forall x)(zx \Rightarrow Ax)) \wedge (Bi\ Ex)((\mu B)z)),$$

B: Few B are A ($:=$ Almost all B are not A) $:= Q^{\forall}_{Bi\,Ex}(B, \neg A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) \,\&\, (\forall x)(zx \Rightarrow \neg Ax)) \wedge (Bi\ Ex)((\mu B)z)),$$

T: Most B are $A := Q^{\forall}_{Bi\,Ve}(B, A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) \,\&\, (\forall x)(zx \Rightarrow Ax)) \wedge (Bi\ Ve)((\mu B)z)),$$

D: Most B are not $A := Q^{\forall}_{Bi\,Ve}(B, \neg A) \equiv$

$$(\exists z)((\Delta(z \subseteq B) \,\&\, (\forall x)(zx \Rightarrow \neg Ax)) \wedge (Bi\ Ve)((\mu B)z)),$$

"Many","More then half", "Some"

F: More then half B are $A := Q^{\forall}_{Bi\,VR}(B,A) \equiv$
$$(\exists z)((\boldsymbol{\Delta}(z \subseteq B)\,\&(\forall x)(zx \Rightarrow Ax)) \wedge (\,Bi\,VR)((\mu B)z)),$$

V: More then half B are not $A := Q^{\forall}_{Bi\,VR}(B,\neg A) \equiv$
$$(\exists z)((\boldsymbol{\Delta}(z \subseteq B)\,\&(\forall x)(zx \Rightarrow \neg Ax)) \wedge (\,Bi\,VR)((\mu B)z)),$$

K: Many B are $A := Q^{\forall}_{\neg(Sm\,\bar{\boldsymbol{\nu}})}(B,A) \equiv$
$$(\exists z)((\boldsymbol{\Delta}(z \subseteq B)\,\&(\forall x)(zx \Rightarrow Ax)) \wedge \neg(Sm\,\bar{\boldsymbol{\nu}})((\mu B)z)),$$

G: Many B are not $A := Q^{\forall}_{\neg(Sm\,\bar{\boldsymbol{\nu}})}(B,\neg A) \equiv$
$$(\exists z)((\boldsymbol{\Delta}(z \subseteq B)\,\&(\forall x)(zx \Rightarrow \neg Ax)) \wedge \neg(Sm\,\bar{\boldsymbol{\nu}})((\mu B)z)),$$

I: Some B are $A := Q^{\exists}_{Bi\boldsymbol{\Delta}}(B,A) \equiv (\exists x)(Bx \wedge Ax)$,

O: Some B are not $A := Q^{\exists}_{Bi\boldsymbol{\Delta}}(B,\neg A) \equiv (\exists x)(Bx \wedge \neg Ax)$.

Syllogism, validity

- A syllogism denoted by $\langle P_1, P_2, C\rangle$ is a kind of logical argument in which the *conclusion* C is inferred from two *premises* — *major* P_1 and *minor* P_2.
- By intermediate syllogism we mean traditional syllogism where we replace one or more of its formulas with some containing intermediate quantifiers.
- The syllogism is strongly valid if $T^{\mathrm{IQ}} \vdash P_1 \,\&\, P_2 \Rightarrow C$, or equivalently, if $T^{\mathrm{IQ}} \vdash P_1 \Rightarrow (P_2 \Rightarrow C)$

Classification of IGS

Suppose that Q_1, Q_2, Q_3 are intermediate quantifiers and $X, Y, M \in \mathit{Form}_{o\alpha}$

Figure I

$$\frac{\begin{array}{c} Q_1\ M \text{ is } Y \\ Q_2\ X \text{ is } M \end{array}}{Q_3\ X \text{ is } Y}$$

Figure II

$$\frac{\begin{array}{c} Q_1\ Y \text{ is } M \\ Q_2\ X \text{ is } M \end{array}}{Q_3\ X \text{ is } Y}$$

Figure III

$$\frac{\begin{array}{c} Q_1\ M \text{ is } Y \\ Q_2\ M \text{ is } X \end{array}}{Q_3\ X \text{ is } Y}$$

Figure IV

$$\frac{\begin{array}{c} Q_1\ Y \text{ is } M \\ Q_2\ M \text{ is } X \end{array}}{Q_3\ X \text{ is } Y}$$

Example of strongly valid syllogism of Figure I

ATT-I:
P_1:All women are well dressed
P_2:Most people in the party are women
C:Most people in the party are well dressed

The syllogism above is strongly valid. This means that if there is a model $\mathcal{M} \models T^{\mathsf{IQ}}$ such that $\mathcal{M}(P_1) = a$ and $\mathcal{M}(P_1) = b$ then $a \otimes b \leq \mathcal{M}(C)$.

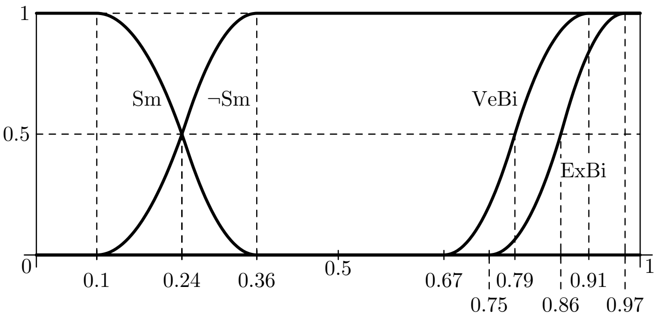

Figure: Shapes of the extensions of evaluative expressions in the context $[0, 1]$ used in the example above.

Valid implications

Valid implications in T^{IQ}

(a) $T^{\mathsf{IQ}} \vdash \mathbf{A} \Rightarrow \mathbf{P}$, $T^{\mathsf{IQ}} \vdash \mathbf{P} \Rightarrow \mathbf{T}$, $T^{\mathsf{IQ}} \vdash \mathbf{T} \Rightarrow \mathbf{F}$,
$T^{\mathsf{IQ}} \vdash \mathbf{F} \Rightarrow \mathbf{K}$.

(a) $T^{\mathsf{IQ}} \vdash \mathbf{E} \Rightarrow \mathbf{B}$, $T^{\mathsf{IQ}} \vdash \mathbf{B} \Rightarrow \mathbf{D}$, $T^{\mathsf{IQ}} \vdash \mathbf{D} \Rightarrow \mathbf{V}$,
$T^{\mathsf{IQ}} \vdash \mathbf{V} \Rightarrow \mathbf{G}$.

Valid implications with presupposition in T^{IQ}

(a) $T^{\mathsf{IQ}} \vdash {}^*\mathbf{A} \Rightarrow \mathbf{I}$, $T^{\mathsf{IQ}} \vdash {}^*\mathbf{P} \Rightarrow \mathbf{I}$, $T^{\mathsf{IQ}} \vdash {}^*\mathbf{T} \Rightarrow \mathbf{I}$,
$T^{\mathsf{IQ}} \vdash {}^*\mathbf{F} \Rightarrow \mathbf{I}$, $T^{\mathsf{IQ}} \vdash {}^*\mathbf{K} \Rightarrow \mathbf{I}$.

(b) $T^{\mathsf{IQ}} \vdash {}^*\mathbf{E} \Rightarrow \mathbf{O}$, $T^{\mathsf{IQ}} \vdash {}^*\mathbf{B} \Rightarrow \mathbf{O}$, $T^{\mathsf{IQ}} \vdash {}^*\mathbf{D} \Rightarrow \mathbf{O}$,
$T^{\mathsf{IQ}} \vdash {}^*\mathbf{V} \Rightarrow \mathbf{O}$, $T^{\mathsf{IQ}} \vdash {}^*\mathbf{G} \Rightarrow \mathbf{O}$.

Affirmative syllogisms of Figure-I.

Let **AAA**, **APP**, **ATT**, **AFF**, **AKK**, **AII** be strongly valid in T^{IQ}.
Then the following syllogisms are strongly valid in T^{IQ}:

AAA					
AAP	**APP**				
AAT	**APT**	**ATT**			
AAF	**APF**	**ATF**	**AFF**		
AAK	**APK**	**ATK**	**AFK**	**AKK**	
A*AI	**A*PI**	**A*TI**	**A*FI**	**A*KI**	**AII**

Negative syllogisms of Figure-I.

Let **EAE**, **EPB**, **ETD**, **EFV**, **EKG**, **EIO** be strongly valid in T^{IQ}. Then the following syllogisms are strongly valid in T^{IQ}:

EAE					
EAB	**EPB**				
EAD	**EPD**	**ETD**			
EAV	**EPV**	**ETV**	**EFV**		
EAG	**EPG**	**ETG**	**EFG**	**EKG**	
E*AO	**E*PO**	**E*TO**	**E*FO**	**E*KO**	**EIO**

Negative syllogisms of Figure-II.

Let **AEE**, **ABB**, **ADD**, **AVV**, **AGG**, **AOO** be strongly valid in T^{IQ}. Then the following syllogisms are strongly valid in T^{IQ}:

$\boxed{\mathbf{AEE}}$					
$\mathbf{AEB}$	$\boxed{\mathbf{ABB}}$				
$\mathbf{AED}$	$\mathbf{ABD}$	$\boxed{\mathbf{ADD}}$			
$\mathbf{AEV}$	$\mathbf{ABV}$	$\mathbf{ADV}$	$\boxed{\mathbf{AVV}}$		
$\mathbf{AEG}$	$\mathbf{ABG}$	$\mathbf{APG}$	$\mathbf{AVG}$	$\boxed{\mathbf{AGG}}$	
$\mathbf{A^*EO}$	$\mathbf{A^*BO}$	$\mathbf{A^*DO}$	$\mathbf{A^*VO}$	$\mathbf{A^*GO}$	$\boxed{\mathbf{AOO}}$

Negative syllogisms of Figure-II.

Let **EAE**, **EPB**, **ETD**, **EFV**, **EKG**, **EIO** be strongly valid in T^{IQ}. Then the following syllogisms are strongly valid in T^{IQ}:

EAE					
EAB	**EPB**				
EAD	**EPD**	**ETD**			
EAV	**EPV**	**ETV**	**EFV**		
EAG	**EPG**	**ETG**	**EFG**	**EKG**	
E*AO	**E*PO**	**E*TO**	**E*FO**	**E*KO**	**EIO**

Relationship between Figure-I and Figure-II.

Let $\langle \mathbf{Q_1}, \mathbf{Q_2} \rangle$ be the following pairs of quantifiers: $\langle \mathbf{A}, \mathbf{E} \rangle$, $\langle \mathbf{A}, \mathbf{B} \rangle$, $\langle \mathbf{A}, \mathbf{D} \rangle$, $\langle \mathbf{A}, \mathbf{V} \rangle$, $\langle \mathbf{A}, \mathbf{G} \rangle$, $\langle \mathbf{P}, \mathbf{B} \rangle$, $\langle \mathbf{P}, \mathbf{D} \rangle$, $\langle \mathbf{P}, \mathbf{V} \rangle$, $\langle \mathbf{P}, \mathbf{G} \rangle$, $\langle \mathbf{T}, \mathbf{D} \rangle$, $\langle \mathbf{T}, \mathbf{V} \rangle$, $\langle \mathbf{T}, \mathbf{G} \rangle$, $\langle \mathbf{F}, \mathbf{V} \rangle$, $\langle \mathbf{F}, \mathbf{G} \rangle$, $\langle \mathbf{K}, \mathbf{G} \rangle$. Then every negative syllogism $\mathbf{EQ_1Q_2}$-I is strongly valid in T^{IQ} if and only if $\mathbf{EQ_1Q_2}$-II is strongly valid in T^{IQ}.

Let $^*\mathbf{Q}$ be one of the following quantifiers with presupposition: $^*\mathbf{A}$, $^*\mathbf{P}$, $^*\mathbf{T}$, $^*\mathbf{F}$, $^*\mathbf{K}$. Then every negative syllogism $\mathbf{E}{}^*\mathbf{QO}$-I is strongly valid in T^{IQ} if and only if $\mathbf{E}{}^*\mathbf{QO}$-II is strongly valid in T^{IQ}.

Definition of a theory $T[B, B']$

Let $B, B' \in \mathit{Form}_{o\alpha}$. The theory $T[B, B']$ is a consistent extension of of T^{IQ} such that

(a) $T[B, B'] \vdash B \equiv B'$,

(b) $T[B, B'] \vdash (\exists x_\alpha)\mathbf{\Delta} Bx$ and $T[B, B'] \vdash (\exists x_\alpha)\mathbf{\Delta} B'x$.

Affirmative syllogisms of Figure-III.

Let $\mathbf{AII}, \mathbf{IAI}$ be strongly valid in T^{IQ} and $\mathbf{PKI}, \mathbf{TFI}, \mathbf{FTI}, \mathbf{KPI}$, be strongly valid in $T[B, B']$. Then the following is true:

(a) all the syllogisms denoted by the dashed line are strongly valid in $T[B, B']$-non-trivial syllogisms,

(b) the others syllogisms are strongly valid in T^{IQ}.

*AAI	*PAI	*TAI	*FAI	*KAI	IAI
A*PI	PPI	TPI	FPI	KPI	
A*TI	PTI	TTI	FTI		
A*FI	PFI	TFI			
A*KI	PKI				
AII					

Negative syllogisms of Figure-III.

Let **EIO**, **OAO** be strongly valid in T^{IQ} and
BKO, **DFO**, **VTO**, **GPO**, be strongly valid in $T[B, B']$.

(a) all the syllogisms denoted by the dashed line are strongly valid in $T[B, B']$-non-trivial syllogisms,

(b) the others syllogisms are strongly valid in T^{IQ}.

E*AO	***BAO**	***DAO**	***VAO**	***GAO**	**OAO**
E*PO	**BPO**	**DPO**	**VPO**	**GPO**	
E*TO	**BTO**	**DTO**	**VTO**		
E*FO	**BFO**	**DFO**			
E*KO	**BKO**				
EIO					

All syllogisms of Figure-IV.

Using the main three classical syllogisms **IAI**-IV, **AEE**-IV and **EIO**-IV we can prove strong validity of all the intermediate generalized syllogisms from Figure-IV.

*AAI	AEE	E*AO
*PAI	AEB	E*PO
*TAI	AED	E*TO
*FAI	AEV	E*FO
*KAI	AEG	E*KO
IAI	A*EO	EIO

Relationships between Figure-III and Figure-IV.

Let ${}^*\mathbf{Q}$ be the following quantifiers with presupposition: ${}^*\mathbf{A}, {}^*\mathbf{P}, {}^*\mathbf{T}$, ${}^*\mathbf{F}$, ${}^*\mathbf{K}$. Then every negative syllogism $\mathbf{E}{}^*\mathbf{QO}$-III is strongly valid in T^{IQ} if and only if $\mathbf{E}{}^*\mathbf{QO}$-IV is strongly valid in T^{IQ}.

Results

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- We introduced new intermediate generalized quantifier ”more then half“ and also 19 new intermediate generalized syllogisms.
- We found for every Figure-I-IV the main strongly valid syllogisms using them we may prove the strong validity of the all 144 intermediate generalized syllogisms.

Thank you for your attention.