Option pricing

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Introduction

Types of financial market:

- commodity market,
- currency market or foreign exchange market,
- bond market,
- stock market,
- futures and options market.

What is an stock?

Stocks (also known as shares or eqities):

- company needs to raise money,
- sell shares to investors,
- company is "owned" by its shareholders,
- profit is paid out as a dividend.

GOOGLE stock prices

Modelling stock

We assume that the asset price S follows a geometric Brownian motion

$$\mathrm{d}S = (\rho - q)S\mathrm{d}t + \sigma S\mathrm{d}W, \tag{1}$$

with drift $\rho > 0$ and standard deviation $\sigma > 0$. The presence of dividends is described by a dividend yield $q \geq 0$ and W is standard Wiener proces.

What is an option?

Let us make an option contract.

Simple example: A call option

Call option is *right* to **buy** an share.

- Contract starts today.
- On 12 May 2013 the holder of the option *may* buy one share for 614€.
- Two possible situations that might occur od the expiry date....

Simple example: A call option

Holder of the option *may* buy one share for 614€.

Two possible situations that might occur on the expiry date:

1. share price is 624 €,
2. share price is 604 €.

Simple example: A call option

Holder of the option *may* buy one share for 614€.

Two possible situations that might occur on the expiry date:

1. share price is 624 €, holder exercise the option,
2. share price is 604 €, would not be sensible to exercise.

Profit 10 €.

Model

Black–Scholes partial differential equation:

$$\partial_t V + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V + (r-q) S \partial_S V - rV = 0, \tag{2}$$

Terminal pay-off conditions:

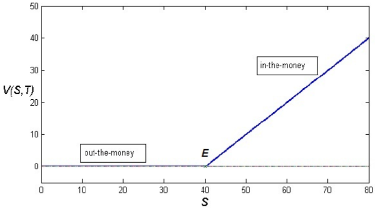
1. European call option:

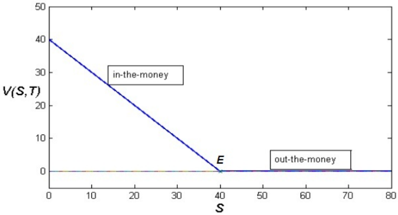
$$V(S,T) = \max(S-E, 0), \tag{3}$$

2. European put option:

$$V(S,T) = \max(E-S, 0). \tag{4}$$

Terminal pay-off conditions

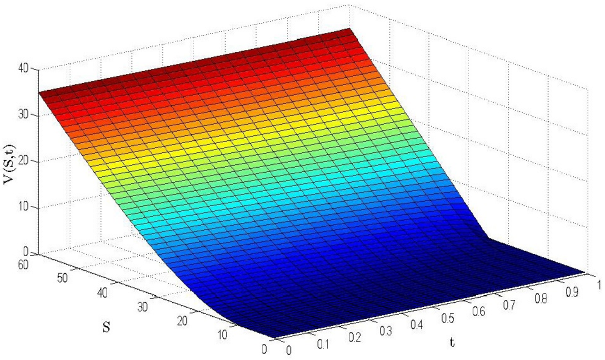




european call option:
$$V(S,T) = \max(S - E, 0)$$

european put option:
$$V(S,T) = \max(E - S, 0)$$

Solution



European call option

The non-linear Black-Scholes model

Motivation for studying

Taking into account e.g.:

- nontrivial transaction costs,

Motivation for studying

Taking into account e.g.:

- nontrivial transaction costs,
- investor's preferences,
- feedback and illiquid market effects,
- risk from a volatile (unprotected) portfolio.

The non-linear Black-Scholes model

$$\partial_t V + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V + (r-q) S \partial_S V - rV = 0, \tag{5}$$

$$\sigma = \sigma(S^2 \partial_S^2 V, S, T-t).$$

Terminal pay-off conditions:

- European call option:

$$V(S,T) = \max(S-E, 0), \tag{6}$$

- European put option:

$$V(S,T) = \max(E-S, 0). \tag{7}$$

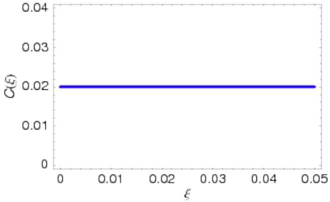
The non-linear Black-Scholes model

- new model
- based on original RAPM

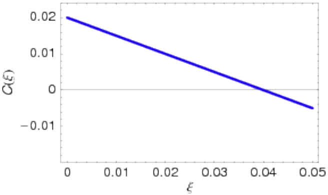
The Risk–adjusted pricing methodology with variable transaction costs

Idea: transaction costs - decreasing konvex function of the amount of traded asstets

The functions of transaction costs



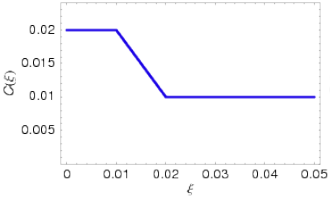
a)Constant: $C(\xi) = \bar{C}$



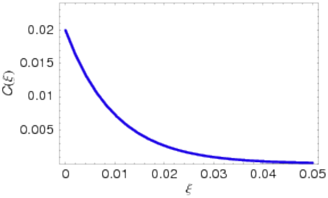
b)Linear decreasing: $C(\xi) = \bar{C} - \kappa\xi$

Amster model

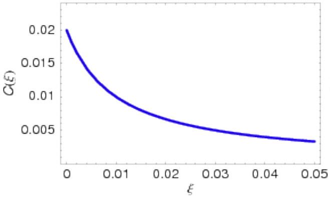
New functions of transaction costs



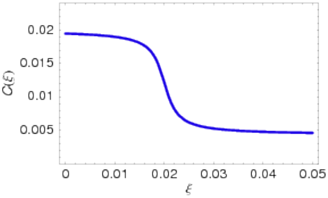
c) $C(\xi)$ Piecewise linear function



d) Exponential: $C(\xi) = \bar{C}\exp(-\kappa\xi)$



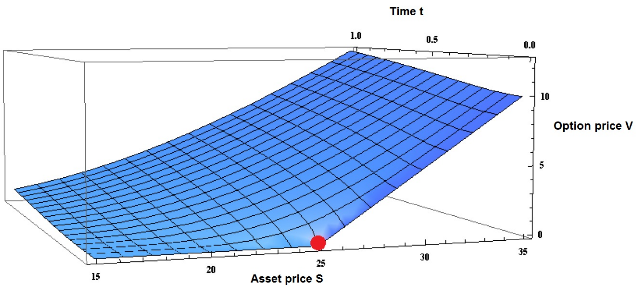
e) $C(\xi) = \bar{C}/(1+\kappa\xi)$



f) $C(\xi) = \bar{C} - \delta(\arctan(\kappa(\xi-\xi_1)) + \pi/2)$

Conclusion

- types of financial markets, stocks and options
- option pricing models of Black–Scholes type
- more realistic non–linear B–S models
- transaction costs
- numerical methods



Thank you for your attention.