On fuzzy topogeneous orders on powersets of fuzzy sets

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ISCAMI 2012
May 10 - May 13, 2012,
Malenovice, Czech Republic

Topology, Uniformity and Proximity

Three important general topology categories:

Topological spaces and continuous mappings

Topology: $T \subseteq 2^X$ such that

1. $\emptyset, X \in T$;
2. $U, V \in T \Rightarrow U \cap V \in T$;
3. $U_i \in T \forall i \in I \Rightarrow \bigcup_i U_i \in T$

(X, T) and (Y, T') topological spaces. Mapping f from X to Y is called continuos, if $f^{-1}(U) \in T$ for all $U \in T'$.

Topology, Uniformity and Proximity

Proximity spaces and proximally continuous mappings

Proximity: $\delta \subseteq 2^X \times 2^X$ such that

1. $(\emptyset, X) \notin \delta$
2. $(A, B) \in \delta \iff (B, A) \in \delta$;
3. $(A, B \cup C) \in \delta \iff (A, B) \in \delta$ or $(A, C) \in \delta$
4. $(A, B) \notin \delta$ then $\exists C, D$: $(A, C) \notin \delta$, $(B, D) \notin \delta$ and $C \cup D = X$.

(X, δ) and (Y, δ') proximity spaces. Mapping f from X to Y is called proximally continuous with respect to δ and δ', if from $(A, B) \in \delta$ $(A, B \subset X)$ follows $(f(A), f(B)) \in \delta'$ $(f(A), f(B) \subset Y)$.

Topology, Uniformity and Proximity

$V \subset X \times X$ an entourage of diagonal, if $\Delta \subset V$ and $V = -V$, where $\Delta = \{(x, x) : x \in X\}$ and $-V = \{(x, y) : (y, x) \in V\}$.
$\mathcal{D}_X$ family of all entourages of the diagonal.

Uniform space and uniformly continuous mappings

Uniformity: $\mathcal{U} \subseteq \mathcal{D}_X$ such that

1. If $V \in \mathcal{U}$ and $V \subset W \in \mathcal{D}_X$, then $W \in \mathcal{U}$
2. If $V_1, V_2 \in \mathcal{U}$, then $V_1 \cap V_2 \in \mathcal{U}$
3. For every $V \in \mathcal{U}$ exists $W \in \mathcal{U}$ such that $2W \subset V$
4. $\bigcap \mathcal{U} = \Delta$.

$(X, \mathcal{U})$ and $(Y, \mathcal{V})$ uniform spaces. Mapping f from X to Y is called uniformly continuous with respect to $\mathcal{U}$ and $\mathcal{V}$, if for every $V \in \mathcal{V}$ exists $U \in \mathcal{U}$ such that for all $x, x' \in X$ it holds $(f(x), f(x')) \in V$, when ever $(x, x') \in U$

Syntopogeneous structures in Topology

Why we need syntopogeneous structures

Syntopogeneous structure is a concept which allows to develop a unified approach to all three topological categories:

- Topological spaces and continuous mappings;
- Uniform spaces and uniformly continuous mappings;
- Proximity spaces and proximally continuous mappings.

Syntopogeneous structures were introduced by A. Csaszar in 1963.

Topogeneous orders (in crisp case)

Semi topogeneous order

Semi-topogeneous order on a set X is a relation σ on it powerset 2^X such that

- $(\emptyset, \emptyset), (X, X) \in \sigma$
- If $M' \leq M$ and $N \leq N'$ and $(M, N) \in \sigma$ then $(M', N') \in \sigma$.
- If $(M, N) \in \sigma$ then $M \subseteq N$

Topogeneous order

Semi-topogeneous order on a set X is called a topogeneous order if

- $(M_1 \cup M_2, N) \in \sigma \iff (M_1, N), (M_2, N) \in \sigma$.
- $(M, N_1 \cap N_2) \in \sigma \iff (M, N_1), (M, N_2) \in \sigma$.

Syntopogeneous structures

Syntopogeneous structures

A family $\mathcal{S}$ of topogeneous orders on a set X is called a syntopogeneous structure if

1. $\mathcal{S}$ is directed, that is
   $\sigma_1, \sigma_2 \in \mathcal{S} \Longrightarrow \exists \sigma \in \mathcal{S}$ such that $\sigma_1 \cup \sigma_2 \subseteq \sigma$;
2. $\forall \sigma \in \mathcal{S}\ \exists \sigma' \in \mathcal{S}$ such that $\sigma' \circ \sigma' \supseteq \sigma$,
   where $(\mathrm{M}, \mathrm{N}) \in \sigma_1 \circ \sigma_2$, if $\exists \mathrm{P} \in 2^{\mathrm{X}}$ such that $(\mathrm{M}, \mathrm{P}) \in \sigma_1$ and $(\mathrm{P}, \mathrm{N}) \in \sigma_2$.

Situation in fuzzy Topology

In fuzzy topology we have developed theories of

1. Fuzzy topologies;
2. Fuzzy proximities;
3. Fuzzy uniformities

Problem

Find the appropriate concepts for fuzzy syntopogeneous structures.

Approach to the fuzzy version

How to define fuzzy semi-topogeneous order

L-fuzzy semi-topogeneous order on a set X is a L-fuzzy relation σ on its L powerset L^{X}, that is $\sigma : \mathrm{L}^{\mathrm{X}} \times \mathrm{L}^{\mathrm{X}} \to \mathrm{L}$ such that

- $\sigma(0_{\mathrm{L}}, 0_{\mathrm{L}}) = \sigma(1_{\mathrm{L}}, 1_{\mathrm{L}}) = 1_{\mathrm{L}}$
- If $\mathrm{M}' \leq \mathrm{M}$ and $\mathrm{N} \leq \mathrm{N}'$ then $\sigma(\mathrm{M}, \mathrm{N}) \leq \sigma(\mathrm{M}', \mathrm{N}')$.
- If $\sigma(\mathrm{M}, \mathrm{N})$ then $\mathrm{M} \tilde{\subseteq} \mathrm{N}$

- As the substitute for the last property we take

$$\sigma(\mathrm{M}, \mathrm{N}) \leq \mathrm{M} \tilde{\subseteq} \mathrm{N}$$

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- $\sigma(0_L, 0_L) = \sigma(1_L, 1_L) = 1_L$
- If $M' \leq M$ and $N \leq N'$ then $\sigma(M, N) \leq \sigma(M', N')$.
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- Next problem

How to define fuzzy inclusion ?

$$M \tilde{\subseteq} N$$

- Inclusion of ordinary sets

In crisp case $M \subset N$

For all $x \in X$ if $x \in M$ then $x \in N$

- Realization of this idea in fuzzy case

Realization in fuzzy case

$$M \tilde{\subseteq} N = \inf_{x \in X} (M(x) \mapsto N(x))$$

where $\mapsto$ is an implicator on L

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Implicator generally is a mapping

$$\mapsto \mathrm{L} \times \mathrm{L} \to \mathrm{L}$$

satisfying certain conditions which are extracted from the basic properties of an Implication in classical logic. However different authors axiomatize different properties. There is done much work comparing different properties taken in the definition of an implicator. In particular:
S. Gotwald: Many-valued logic, Chapter I in Mathematics of fuzzy sets: Logic, Topology and Measure Theory, Kluwer Acad. Publ. 1999; S. Gotwald, Mehrwertige Logic: Eine Einfurung in Theorie und Anwendungen, Akademie Verlag, Berlin, 1989.

Implicator

For our merits we take the following axioms for $\mapsto L \times L \to L$:

- $a \mapsto b$ is non-increasing on the first argument;
- $a \mapsto b$ is non-decreasing on the second argument;
- $0 \mapsto a = 1_L$ for every $a \in L$ (left boundary condition);
- $1 \mapsto a = a$ for every $a \in L$ (left neutrality)
- $(a \mapsto 0) \mapsto (b \mapsto 0) = b \mapsto a$.

- Remark: Note that properties (1) - (4) are assumed (as far as we know) by most researches in this subject, while (5) is specific for our merits.
- Remark: From (5) and (4) we have the following important double negation property: $(a \mapsto 0) \mapsto 0 = a$ for every $a \in L$. Thus $a \mapsto 0$ is an order reversing involution and we write $a^c = a \mapsto 0$.

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Implicator

Examples of appropriate implicators

- Implication $\mapsto 2 \times 2 \to 2$ in the classical logic;
- If $(L, \wedge, \vee, *)$ is an MV-algebra and $\mapsto$ is the corresponding residuation.
- In particular if $L = [0, 1]$ with Łukasiewcz conjunction $*$ the corrersponding residium is implication:

$$a \mapsto b = \max\{1 - a + b, 0\}.$$

- If $L = [0, 1]$ and $a \mapsto b$ is Kleene-Dienes implication:

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Fuzzy semi-topogeneous orders

Thus let X be a set, L a complete lattice and $\mapsto : L \times L \to L$

Semi-topogeneous order on a set X is a L-fuzzy relation σ on its L-powerset L^X, that is $\sigma : L^X \times L^X \to L$ such that

(1to) $\sigma(0_L, 0_L)) = \sigma(1_L, 1_L) = 1_L$

(2to) If $M' \leq M$ and $N \leq N'$ then $\sigma(M, N) \leq \sigma(M', N')$.

(3to) $\sigma(M, N) \leq M \tilde{\subseteq} N$ where $M \tilde{\subseteq} N = \inf_{x \in X}(M(x) \mapsto N(x))$, where $\mapsto$ is an implicator.

Special properties of fuzzy semi-topogeneous orders

- Fuzzy semitopogeneous order is called topogeneous if

(4to) $\sigma(M_1 \vee M_2, N) = \sigma(M_1 N) \wedge \sigma(M_2, N)$.
(5to) $\sigma(M, N_1 \wedge N_2) = \sigma(M, N_1) \wedge \sigma(M, N_2)$

- Fuzzy topogeneous order is called perfect if

(6to) $\sigma(\bigvee_i M_i, N) = \bigwedge_i \sigma(M_i, N)$.

- Fuzzy topogeneous order is called biperfect if it is perfect and

(7to) $\sigma(M, \bigwedge_i N_i) = \bigwedge_i \sigma(M, N_i)$

- Fuzzy semitopogeneous order is called symmetric if

(8to) $\sigma(M, N) = \sigma(N^c, M^c)$

L-fuzzy syntopogeneous structures

Definition

An L-fuzzy syntopogeneous structure on a set X is a family $\mathcal{S}$ of L-fuzzy topogeneous orders on X such that

- $\mathcal{S}$ is directed, that is given two L-fuzzy topogeneous orders $\sigma_1, \sigma_2 \in \mathcal{S}$ there exists $\sigma \in \mathcal{S}$ such that $\sigma_1 \vee \sigma_2 \leq \sigma$;
- For every $\sigma \in \mathcal{S}$ there exists $\sigma' \in \mathcal{S}$ such that $\sigma \leq \sigma' \circ \sigma'$, where $\sigma_1 \circ \sigma_2 = \{\bigvee(\sigma_1(\mathrm{M},\mathrm{P}) \wedge \sigma_2(\mathrm{P},\mathrm{N}) : \mathrm{P} \in \mathrm{L}^{\mathrm{X}}\}$.

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Comments regarding the position of our research

- If $L = 2$ and $\mapsto$ is the classical implication, then we obtain A. Csaszar's syntopogeneous structures (crisp-crisp)
  A. Csaszar, Foundations of General Topology, Pergamon Press, 1963.
- If $\mapsto [0,1] \times [0,1] \to 2$ we obtain Katsaras-Petalas syntopogeneous structure (fuzzy-crisp)
  A.K. Katsaras, C.G Petalas, On fuzzy syntopogeneous structures, J. Math. Anal. Appl., 99 (1984), 219-236.
- If $\mapsto [0,1] \times [0,1] \to [0,1]$ Łukasiewicz implication (fuzzy-fuzzy)
  A.Š. Fuzzy Syntopogeneoius structures, Quaestiones Math., 20 (1997), 431-461
- New: fuzzy syntopogeneous structures based on different implicators (L_1-fuzzy - L_2-fuzzy).

L-fuzzy topologies and perfect L-fuzzy topogeneous orders

Theorem

Let $\sigma : \mathrm{L}^{\mathrm{X}} \times \mathrm{L}^{\mathrm{X}} \to \mathrm{L}$ be a perfect topogeneous fuzzy order. Then the mapping : $\mathcal{T} : \mathrm{L}^{\mathrm{X}} \to \mathrm{L}$ defined by $\mathcal{T}_{\sigma}(\mathrm{M}) = \sigma(\mathrm{M}, \mathrm{M}), \mathrm{M} \in \mathrm{L}^{\mathrm{X}}$ is an L-fuzzy topology. Conversely, given an L-fuzzy topology $\mathcal{T} : \mathrm{L}^{\mathrm{X}} \to \mathrm{L}$ on X, the mapping $\sigma_{\mathcal{T}} : \mathrm{L}^{\mathrm{X}} \times \mathrm{L}^{\mathrm{X}} \to \mathrm{L}$ defined by the equality

$$\sigma_{\mathcal{T}}(\mathrm{M}, \mathrm{N}) = \bigvee \{\mathcal{T}(\mathrm{P}) : \mathrm{M} \leq \mathrm{P} \leq \mathrm{N}, \mathrm{P} \in \mathrm{L}^{\mathrm{X}}\}$$

is a perfect topogeneous fuzzy order. Besides $\mathcal{T}_{\sigma_{\mathcal{T}}} = \mathcal{T}$ and $\sigma_{\mathcal{T}_{\sigma}} = \sigma$ for every L-fuzzy topology $\mathcal{T}$ and every perfect L-fuzzy topogeneous order σ.

L-fuzzy proximities and L-fuzzy symmetric topogeneous orders

Theorem

Let $\sigma : L^X \times L^X \to L$ be a symmetric L-fuzzy topogeneous order on X. Then the mapping $\delta_\sigma : L^X \times L^X \to L$ defined by

$$\delta(A, B) = \sigma(A, B^c) \mapsto 0$$

is an L-fuzzy proximity on X. Conversely, let $\delta : L^X \times L^X \to L$ be an L-fuzzy proximity. Then with following equality we gain

$$\sigma(A, B) = \delta(A, B^c) \mapsto 0$$

a symmetric L-fuzzy topogeneous order on X. Besides $\delta_{\sigma_\delta} = \delta$ and $\sigma_{\delta_\sigma} = \sigma$ for every symmetric L-fuzzy topogeneous order σ and for any L-fuzzy proximity δ.

Some directions for further research

- Find a natural bijection between the family of all L-fuzzy uniformities on a set X and the set of all biperfect L-fuzzy syntopogeneous structures.
- Develop categorical framing of the theory of L-fuzzy syntopogeneous structures
- Analyse relations between categories of syntopogeneous structures for different implicators
- Develop the theory of L-fuzzy syntopogeneous structures for varied lattices L in order to be coherent with variable-bases fuzzy topologies.

Thank you for your attention!