Powerset operators induced by fuzzy relations as a basis for fuzzification of various mathematical structures

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One of the most important concepts in mathematics of fuzzy structures is the concept of a fuzzy relation first introduced by Zadeh in [5] and now studied and applied in a huge number of publications in various fields of research. Working with fuzzy relations naturally leads to the prei-mages and images of fuzzy sets under fuzzy relations and in the long run to forward and backward operators on fuzzy powersets. Along with purely theoretical, in particulary, within the framework of category theory, work done in the study of such operators (see, e.g. papers by S.E. Rodabaugh, S. Solovyov, J. Močkor et al) forward and backward operators play fundamental role also as a tool for fuzzification of various mathematical theories.

In first part of the talk we present a unified approach to the theory of forward and backward operators induced by fuzzy relations paying special attention to their algebraic and topological properties. In the second part we illustrate the fundamental role of these operators in the fuzzification process of the three important branches of mathematics that emerged in the last quarter of the pevious century: the theory of rough sets whose origins were laid by Z. Pawlak [1], mathematical morphology initiated by J. Serra [3] and formal concept analysis founded by R. Wille [4]. Also in these special cases we will emphasize the topological constituent of the construction.

References

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