CATEGORIC SIMULATION OF PRODUCTION FLOWS

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Joint work with David Kruml.

In this lecture we develop a notion of *categoric simulation* of production flows. This means that we want to see simulation as an abstract mathematical structure - a *category* - consisting of *objects* - places that each processed unit passes at an accurate time, and *morphisms* - devices such as production processes, transfers or stacks. Simulation has been successfully adopted in numerous studies related to manufacturing system design and operation. With simulation, it is also possible to analyze several scenarios and to consider a wide range of performance measures. Categoric simulation is a highly flexible tool which enables us to evaluate effectively different alternatives of system configurations and operating strategies to support decision making in the manufacturing context.

Our categoric model provides the structure of graphs as e.g. in [5] or [7] but adds a *hierarchy* of composed devices. The rationale for developing such a framework is similar to the category-theoretic approach to quantum theory based on [1]. As the starting point of quantum theory, usually the seminal work [2] is cited. The idea of Birkhoff and von Neumann has been to gain an understanding of quantum physics from the perspective of a particular non-classical "logic". The primary motivation of Abramsky and Coecke was in this case rather closely related to actual physical phenomena, the aim is to construct a simplified, technically lightweight description of quantum algorithms and protocols. The category-theoretic approach focuses on the processes and the composition of systems as central notions. In our lecture we demonstrate that the formalism designed to model quantum circuits can well be used to model classical processes — objects and morphisms describe the flow of events, and the tensor product corresponds to parallel processes.

Our approach is, generally speaking, within the tradition of using category theory as a modern language for the description of networks.

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Examples come from programming as in [4], quantum protocols as in [1], electrical circuits as in [3] or Petri nets as in [6].

The type of category suitable in this context is exactly what has been proposed in the context of quantum protocols: dagger compact symmetric monoidal. It is consequently this framework that we may use for the application that we have in mind: we can simulate the production flow in manufacturing. The reason of doing so is actually of a practical nature: even though the data with which we deal is finite, it is too large to allow for straightforward computations. Hence we need ways how to simplify the complexity of composed devices and to this end the categorical approach has proved to be quite useful.

Our research project develops both a *complete methodology* and a *software program* for a direct simulation of production flow in manufacturing. The purpose of the methodology is to support simulation engineers to structure their projects and provide helpful experiences for robust modeling, verification, validation, and analysis of the production flow, using structured modeling and transparent code. The target application for the intended methodology is a detailed modeling of manufacturing systems. The production engineers that use the results will obtain a detailed and real time information on how they can improve their area of the production flow and processes.

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