

ON THE ENTROPY CONJECTURE OF MARCY BARGE

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I shall discuss a positive solution to the following problem, obtained in a joint work with J. Činč and P. Oprocha.

Question (M. Barge, 1989 [Le89]) Does there exist, for every $r \in [0, \infty]$, a pseudo-arc homeomorphism whose topological entropy is r ?

Until now all known pseudo-arc homeomorphisms have had entropy 0 or ∞ . Recall that the pseudo-arc is a compact and connected space (continuum) first constructed by Knaster in 1922 [Kn]. It can be seen as a pathological fractal. According to the most recent characterization [HO18] it is topologically the only, other than the arc, continuum in the plane homeomorphic to each of its proper subcontinua. The pseudo-arc is homogeneous [Bi] and played a crucial role in the classification of homogeneous planar compacta [HO16]. Lewis showed that for any n the pseudo-arc admits a period n homeomorphism that extends to a rotation of the plane, and that any P -adic Cantor group action acts effectively on the pseudo-arc [Le83] (see also [To]). We adapt Lewis' inverse limit constructions, by combining them with a Denjoy-Rees scheme [BCL] (see also [Re], [BKLO]). The positive entropy homeomorphisms that we obtain are periodic point free, except for a unique fixed point.

I shall start my talk by reviewing the role that the pseudo-arc have played in various areas of mathematics, including topology, surface dynamics, complex analysis, isometric theory of Banach spaces and logic, and then will move on to the history of the problem, to conclude with a discussion of its solution.

References

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