University of Ostrava

**Institute for Research and Applications of Fuzzy Modeling**

# A Fuzzy Logic Model of Detective Reasoning

Antonín Dvořák and Vilém Novák

**Research report No. 99**

2006

*Submitted/to appear:*

Proceedings of IPMU2006 Conference

*Supported by:*

Grant 201/04/1033 of the Czech Grant Agency and the research scheme MSM 6198898701 of the MŠMT ČR

**University of Ostrava**
**Institute for Research and Applications of Fuzzy Modeling**
**30. dubna 22, 701 03 Ostrava 1, Czech Republic**

tel.: +420-59-6160234 fax: +420-59-6120 478
e-mail: antonin.dvorak@osu.cz

# A Fuzzy Logic Model of Detective Reasoning*

**Antonín Dvořák and Vilém Novák**

University of Ostrava
Institute for Research and Applications of Fuzzy Modeling
30. dubna 22, 701 03 Ostrava 1, Czech Republic
email: antonin.dvorak@osu.cz

## Abstract

The paper presents a model of reasoning based on a detective story inspired by one episode from famous TV series about Lt. Columbo. It is a demonstration of the power of fuzzy logic in broader sense whose formal frame is fuzzy type theory.

## 1 Introduction

A detective story inspired by one episode of famous TV series about Lt. Columbo represents a typical way of human reasoning. On the ground of it, we demonstrate the power of fuzzy logic in broader sense — an extension of fuzzy type theory (FTT) [7] towards logical theories of evaluating linguistic expressions [9, 10], fuzzy IF-THEN rules [11] and perception-based logical deduction [12].

There are many attempts at translation of natural language into a form more clear and better suited for (automated) deduction. This was originated by Aristotle's theory of syllogisms, and then continued as a red line through all history of logic and linguistics. However, it is important to bear in mind that we are not looking for some hidden ideal structures behind our language. We rather concentrate on features important from the point of view of inferential abilities of natural language and develop formal systems that are models (approximations) of natural language. A repeated drawback of many formalisms is neglect of the *vagueness* phenomenon which, however, is inherently present in natural language semantics (cf. [5]). A recent attempt without this drawback has been originated by L. A. Zadeh who calls it PNL – Precisiated Natural Language [13, 14] (see Section 4).

Below, we will use some ideas of PNL. As a formal frame for our system we use fuzzy type theory (a higher-order fuzzy logic) because our previous experience (cf. [4, 6]) indicates that first-order logical systems are not powerful enough for the proper formalization of natural language.

We must stress that our goal *is not* developing a formal system for automatical solution of criminal cases. Instead, we want to show the power of fuzzy logic and fuzzy type theory on a non-trivial example. We expect that similar reasoning can be used in variety of applications where inherently vague notions play crucial role. For example, in economical analyses we often meet sentences as follows:

> After demanding October, inflation rested in November and prices slightly decreased. Thanks to good weather forecast, prices of oil are decreasing slightly, and dollar strengthens*).

An automated deduction from such statements can be done in a similar way as in our model of detective story.

* The research has been supported by the grant 201/04/1033 of the GA ČR and partially also by the project MSM 6198898701 of the MŠMT ČR.

*) Cited from free economical analysis of Czech Savings Bank.

## 2 Formal tools

### 2.1 Fuzzy type theory

As stated, the main tool for the logical analysis of the detective story is fuzzy type theory. In this section, we will very briefly overview some of its main points. A detailed explanation of FTT can be found in [7]. The classical type theory is in details described in [1].

The *Types* is a set of types constructed iteratively from the atomic types $\epsilon$ (elements) and $o$ (truth values). $Form_\alpha$ denotes a set of formulas of type $\alpha \in Types$. If $A \in Form_\alpha$ is a formula of type $\alpha \in Types$ then we write $A_\alpha$.

Formulas of type $o$ (truth value) can be joined by the following connectives (derived formulas): $\vee$ (disjunction), $\wedge$ (conjunction), **&** (strong conjunction), $\nabla$ (strong disjunction), $\Rightarrow$ (implication). General ($\forall$) and existential ($\exists$) quantifiers are defined as special formulas. For the details about their definition and semantics — see [7].

If $A \in Form_{o\alpha}$ then $A$ represents a property of elements of the type $\alpha$. By abuse of language, we will often say "$A$ is a property" (of elements of type $\alpha$) and similarly, $A_{(o\alpha)\alpha}$ is a relation (between elements of type $\alpha$). We will freely write or omit the type when no misunderstanding may occur.

A theory $T$ is a set of formulas of type $o$ (determined by a subset of special axioms, as usual). Provability is defined as usual.

The operator

$$\imath z_\alpha A_o := \iota_{\alpha(o\alpha)}(\lambda z_\alpha A_o)$$

picks up an element of type $\alpha$ such that the formula $A_o$ is true in the degree 1 for it.

**Semantics.** The structure of truth values in this paper is the Łukasiewicz$_\Delta$ algebra and so, the corresponding FTT is Łukasiewicz (Ł-FTT). $\Delta$ is the Baaz delta [7]. Let $J$ be a language of Ł-FTT. A *frame* for $J$ is a tuple $\mathcal{M} = \langle (M_\alpha, =_\alpha)_{\alpha \in Types}, \mathcal{L}_\Delta \rangle$ where $\mathcal{L}_\Delta$ is Łukasiewicz$_\Delta$ algebra of truth values, $=_\alpha$ is a fuzzy equality on $M_\alpha$.

Recall that if $\beta\alpha$ is a type then the corresponding set $M_{\beta\alpha}$ contains (not necessarily all) functions $f : M_\alpha \longrightarrow M_\beta$.

Let $p$ be an assignment of elements from $\mathcal{M}$ to variables. An interpretation $\mathcal{I}^{\mathcal{M}}$ is a function that assigns every formula $A_\alpha$, $\alpha \in Types$ and every assignment $p$ a corresponding element, that is, a function of the type $\alpha$. A general model is a frame $\mathcal{M}$ such that $\mathcal{I}^{\mathcal{M}}_p(A_\alpha) \in M_\alpha$ holds true.

The following is a special formula representing a non-zero truth value:

$$\Upsilon_{oo} := \lambda z_o \cdot \neg\mathbf{\Delta}(\neg z_o).$$

**Lemma 1**
*If* $T \vdash \Upsilon z_o \,\&\, (z_o \Rightarrow y_o)$ *then* $T \vdash \Upsilon y_o$.

### 2.2 Evaluating linguistic expressions

Because of lack of space and quite complicated formalism, we will only touch this theory and refer to the contribution [9]. All the details can be found in [10].

Evaluating expressions are denoted by $\mathcal{A}, \mathcal{B}$. *Intension* of $\mathcal{A}$ is a formula $\mathrm{Int}(\mathcal{A})$. The formal theory of evaluating expressions is denoted by $T^{Ev}$.

Important role in the theory of evaluating expressions is played by *context*. It characterizes range of possible values for (numerical) variables and is represented by a special type $\omega$. Hence, the type of $\mathrm{Int}(\mathcal{A})$ is $(o\alpha)\omega$. The latter will often be denoted by $\varphi$. For simplicity, we will suppose that in each model, the context is given by a triple $\langle v_L, v_M, v_R \rangle$, where $v_L, v_M, v_R \in \mathbb{R}$ and $v_L < v_M < v_R$. The values $v_L, v_M, v_R$ characterize minimal, middle and maximal value of the given context, respectively. On syntactical level, context is denoted by a variable $w \in Form_\omega$ and represented by three constants $\perp_w, \dagger_w$ and $\top_w$.

We say that the intension $\mathrm{Int}(\mathcal{A}) := A_{(o\alpha)\omega}$ of an evaluating expression $\mathcal{A}$ is *normal* if

$$T^{Ev} \vdash (\forall w)(\exists x)\mathbf{\Delta} A_{(o\alpha)\omega} w x. \qquad (1)$$

Our theory assures that intensions of all evaluating expressions (including their negations) are normal.

### 2.3 Fuzzy IF-THEN rules and perception-based logical deduction

The perception-based logical deduction in the frame of FTT has been described in [8]. Though the method is more general, we will suppose that all considered linguistic expressions are evaluating ones.

A fuzzy IF-THEN rule is a linguistic expression of the form

$$\mathcal{R} := \text{ IF } X \text{ is } \mathcal{A} \text{ THEN } Y \text{ is } \mathcal{B}.$$

where $\mathcal{A}, \mathcal{B}$ are evaluating expressions. *Intension of a fuzzy IF-THEN rule* $\mathcal{R}$ is

$$\mathrm{Int}(\mathcal{R}) := \lambda w\, \lambda w' \cdot \lambda x\, \lambda y \cdot A_{(o\alpha)\omega} wx \Rightarrow \Rightarrow B_{(o\beta)\omega} w'y \quad (2)$$

A *linguistic description* is a set of fuzzy IF-THEN rules. Its *topic* is a set of linguistic expressions $\{\mathrm{Int}(\mathcal{A}_j) \mid j = 1, \ldots, m\}$ and its focus is $\{\mathrm{Int}(\mathcal{B}_j) \mid j = 1, \ldots, m\}$.

In perception-based logical deduction, we must introduce several special formulas. We will give only their informal description and refer to [8] for their precise definitions. The formula $\prec$ denotes the relation of sharpness between (intensions of) evaluating expressions. For example, if $x$ is, at least partly, "very big" in all contexts then it is also "big" in all of them, i.e. $\mathrm{Int}(\textit{very big}) \prec \mathrm{Int}(\textit{big})$. We will also introduce formulas $\textit{Perc}_{(o\varphi)\alpha}$ ($\textit{Perc}\, x_\alpha z_\varphi$ expresses that an intension $z_\varphi$ is a *perception* of $x_\alpha \in \textit{Form}_\alpha$ with respect to the given linguistic description), and $\textit{Eval}_{o(\varphi\alpha\omega)}$ ($\textit{Eval}\ wx\, \mathrm{Int}(\mathcal{A})$ expresses that an element $x$ in context $w$ is evaluated by $\mathcal{A}$).

**Lemma 2**
*(a) Let $Ev_1 \prec Ev_2$. Then*

$$T^{Ev} \vdash \textit{Perc}\ x\, Ev_1 \Rightarrow \textit{Perc}\ x\, Ev_2.$$

*(b) Let $T^{Ev} \vdash z_\varphi wx \Rightarrow z'_\varphi w'y$. Then $T^{Ev} \vdash \textit{Eval}\ \ wxz_\varphi \Rightarrow \textit{Eval}\ \ w'yz'_\varphi$.*

*(c) Let $T \vdash \Upsilon z_o \,\&(z_o \Rightarrow A \,\&\, \neg A)$. Then $T$ is contradictory.*

*(d) Let $T \vdash z_o \,\&\, \Upsilon \neg z_o$ or $T \vdash \Upsilon(z_o \,\&\, \neg z_o)$. In both cases is $T$ contradictory.*

The main theorem of perception-based logical deduction states the following: Let us consider a linguistic description $LD$ such that the fuzzy IF-THEN rules forming it are implications. If we find a formula $\mathrm{Int}(\mathcal{A}_i)$ of an expression from the topic of $LD$ and an element $\mathbf{u}^0$ in the context $\mathbf{w}^0$ such that $A_{\varphi,i}\mathbf{w}^0\mathbf{u}^0$ has a non-zero truth degree then (denoting $\mathbf{b}_i^0 \equiv A_{\varphi,i}\mathbf{w}^0\mathbf{u}^0$) we conclude that the element $\imath y \cdot \mathbf{b}_i^0 \Rightarrow B_{\varphi,i} w'y$ *typical for the formula* $\mathbf{b}_i^0 \Rightarrow B_{\varphi,i} w'y$, is evaluated by the linguistic expression $\mathcal{B}_i$ in every context $w'$, where $\mathrm{Int}(\mathcal{B}_i)$ belongs to the focus of $LD^I$.

**Theorem 1 ([11])**
*Let $A_{(o\alpha)\omega}$, $B_{(o\beta)\omega}$ represent intensions of some evaluating linguistic expressions. Then*

*(a)* $T \vdash (\forall w)(\forall w')(\forall x)(\exists y)\mathbf{\Delta}(A_{(o\alpha)\omega} wx \Rightarrow B_{(o\beta)\omega} w'y).$

*(b)* $T \vdash (\forall w)(\forall w')(\forall x)(\exists y)(\Upsilon\, A_{(o\alpha)\omega} wx \Rightarrow \Upsilon\, B_{(o\beta)\omega} w'y).$

This theorem is somewhat unpleasant consequence of the logical theory of evaluating expressions and the corresponding theory of fuzzy IF-THEN rules. It tells us that given any two evaluating expressions (say, *small* and *big*) then we may prove that if $x$ is evaluated by one implies that $y$ is evaluated by the second (analogous result holds also for their conjunction). In other words, when dealing with evaluating expressions, arbitrary correspondence characterized by a fuzzy IF-THEN rule is provable.

It must be stressed that this result complies with the way how people comprehend natural language. Namely, the theory $T^{Ev}$ characterizes semantics of evaluating linguistic expressions. If fuzzy IF-THEN rules are composed of them then all possible combinations of evaluating expressions have clear meaning and so, if our theory is correct, they (as logical formulas) must be provable. A different question is, whether this corresponds to the reality. For example, the ordinary knowledge when driving a car is: "IF obstacle is near THEN break strongly". It has also a good meaning to say "IF obstacle is near THEN break slightly" but in practice, we never use the latter if we do not want to cause an accident. This leads us to the problem of non-monotonic reasoning (cf. [2]) and namely, to the concept of epistemic state. We deal with a class of theories that themselves are consistent but when using them simultaneously, we may come to contradiction or non-desirable result. Therefore, we must consider also a special preference relation that tells us which theory should be used in the given state (called *belief state* in [2]). At each state, we work in a special theory which in our case is determined by a linguistic description (one or more) and possibly also by some perception (recall that this is a formula). In this paper, we will present this idea only informally.

## 3 The story

Mr. John Smith has been shot dead in his house. He was found by his friend, Mr. Carry. Lt. Columbo suspects Mr. Carry to be the murderer.

Mr. Carry's testimony is the following:

> *I have started from my home at about 6:30, arrived to John's house at about 7, found John dead and went immediately to the phone box to call police. They told me to wait and came immediately.*

Lt. Columbo has found the following evidence about dead Mr. Smith:

> *He had high quality suit with broken wristwatch stopped at 5:45. No evidence of strong strike on his body. Lt. Columbo touched engine of Mr. Carry's car and found it to be more or less cold.*

Lt. Columbo concluded that Mr. Carry lies because of the following: The wristwatch has been broken but high quality wristwatch does not break after not too strong strike. A man having high quality dress and a luxurious house is supposed to have also high quality wristwatch. However, the wristwatch of John Smith is of low quality and so, it does not belong to him. Consequently, it does not show the time of death.

Mr. Carry's car engine is more or less cold, but it must have been hot because he went long (more than about 30 minutes). Therefore, he could not arrive and continue to call the police. He must have stayed there.

Note that there is no direct evidence of Mr. Carry's crime, but:

- Mr. Carry lied about the time of his arrival,
- he could assassinate Mr. Smith, because the time of Mr. Smith's death is unknown.

Hence, Lt. Columbo conjecture, based on indirect evidence, is justified.

## 4 The Method

Our theory can be classified as part of the methodology introduced L. A. Zadeh in his papers [13, 14] and called *Precisiated Natural Language* (PNL). It is an attempt to develop a unified formalism for various tasks involving natural language propositions. A simple application of this methodology to economy has been published in [3].

Two premises of PNL are the following:

(a) Much of the world knowledge is perception based.

(b) Perception based information is intrinsically fuzzy.

The PNL methodology requires existence of the, so-called, World Knowledge Database (WKDB) which contains perception based propositions describing the world knowledge and are used in the deduction process. A *multiagent, modular deduction database* (DDE) contains various rules of deduction.

Our version of PNL incorporates logical machinery. The translation from NL to logical rules is done manually. However, we need not precise specifications of membership functions of fuzzy sets, generalized (fuzzy) quantifiers etc. The reasoning is done mainly on syntactic level using powerful machinery of fuzzy type theory.

The reasoning of Lt. Columbo can be modeled in FLb using combination of logical rules, world knowledge and evidence with the help of the mentioned principles of non-monotonic resoning.

Logical analysis of the story relies on the world knowledge. This contains information about contexts of involved variables, several linguistic descriptions (sets of fuzzy IF-THEN rules) of partial cases, and a set of evidence. The whole story leads to a certain theory of FTT. The involved contexts correspond to some model $\mathcal{M} \models T$. All fuzzy IF-THEN rules as well as the evidence are transformed into special formulas of Ł-FTT and deduction is performed.

Global characterization of the world knowledge:

(i) Characterization of specific *context* interpreted in the considered model. This information comes from experience and knowledge of the world (in a wide sense). For example, we know that the context for heights of people in Europe is always $\langle 40, 165, 220\rangle$ (in cm).

(ii) *Logical rules* are logical theorems of Ł-FTT and theorems given by some considered theory. In our case, this is theory of evaluating linguistic expressions and the logical theory of fuzzy IF-THEN rules.

(iii) *Customs of people.*

(iv) *Properties of products.*

(v) *Knowledge from physics* and other areas of human activity.

Items (iii) and further are characterized by sets of linguistic descriptions consisting of special fuzzy IF-THEN rules acquired by experience. Of course, various further categories can be added depending on the individual situation.

The *evidence* is in our case divided into two subsets:

1. Evidence given by Mr. Carry,
2. evidence found by Lt. Columbo.

Our goal is to show that the conclusions obtained from world knowledge and the evidence found by Lt. Columbo contradict the evidence given by Mr. Carry.

## 5 Description of the World Knowledge and Reasoning

In this section, we will outline how the world knowledge can be formed and provide some basic formalization of it. We will present several linguistic descriptions. Because of the lack of space, we usually write only one specific fuzzy IF-THEN rule in each linguistic description.

### 5.1 World knowledge

(i) *Context:*

  (a) *Drive duration to heat the engine (minutes):* $w_D = \langle 0, 5, 30\rangle$.
  (b) *Temperature of engine (degrees Celsius):* $w_T = \langle 0, 45, 100\rangle$.
  (c) Abstract degrees[†)]: quality, state, strike strength: $\langle 0, 0.5, 1\rangle$.

(ii) *Logical rules:*(see [10], Theorem 6)

$$\textsf{IF } X \textsf{ is } Sm_\nu \textsf{ THEN } X \textsf{ is} \neg Bi \quad (3)$$
$$\textsf{IF } X \textsf{ is } Bi_\nu \textsf{ THEN } X \textsf{ is} \neg Sm \quad (4)$$

where $Sm_\nu, Bi_\nu$ is the intension of "$\nu$ small" ("$\nu$ big") where $\nu$ is some linguistic hedge.

Of course, many other rules based on the logical structure of Ł-FTT and also special theories, such as $T^{Ev}$ are also considered.

†) This is a certain simplification that has been accepted because all the information is subjective and so we may hardly assume that people are able to estimate, e.g. a physical force necessary to break the wristwatch.

(iii) *Knowledge from physics:*

IF *drive duration* is $Bi$ THEN
*engine temperature* is $Bi$ (5)

. . . . . . . . . . . . . . . . . . . . . . . . . . .

(iv) *Customs of people:*

Characterization of relation of quality of clothes depending on the wealth of a person $P$:

IF quality of $P$'s suit is $Bi$ AND
quality of $P$'s house is $Bi$
THEN wealth of $P$ is $Bi$ (6)

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

Various kinds of customs of people can be characterized by *correspondence* which describes relation between two (numerical) domains. It can be informally described as follows: If a value of the first variable is small, and a value of the second variable is also small, then the correspondence of these variables is big. The correspondence is also big, if both variables have medium or big values (note that contexts could be totally different, and that small could mean, e.g. 0.1 for the first variable, and 10000 for the second). If a value of the first variable is small and a value of the second variable is not small, then their correspondence is small, etc. Here it is used for the description of correspondence between person's wealth and the quality of his wristwatch. This informal description can be easily expressed using a linguistic description. Due to the lack of space, it is omitted.

IF wristwatch belongs to $P$ THEN
Corr(quality of $P$'s wristwatch,
wealth of $P$) is $Bi$ (7)

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

(v) *Properties of products:*

IF quality of wristwatch is $Bi$ AND
*strike* is $ML\,Sm$ THEN
state of wristwatch is $Bi$ (8)

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

(High quality wristwatch will not break.)

**Evidence (by Lt. Columbo)**

(Evd1) Touching the engine by hand does not burn; that is, its temperature is about 40°C.

(Evd2) State of wristwatch is zero (the wristwatch is broken).

(Evd3) Quality of suit is $0.9 \in [0,1]$ (very big).

(Evd4) Quality of house is $0.95 \in [0,1]$ (very big).

(Evd5) Strike strength is zero (no evidence of strike on the body).

**Evidence (by Mr. Carry)**

(EvdC1) *My drive duration was about 30 minutes.*

(EvdC2) (*implicit:*) The wristwatch on Mr. Smith's hand belongs to him and it displays time of his death.

### 5.2 Formalization and reasoning

In this section, we will formally demonstrate that Lt. Columbo's conclusion contradicts Mr. Carry's evidence. Our main tool is perception-based logical deduction. As discussed above, we will use the principles of non-monotonic reasoning, that is, we deal with a theory $T$ of Ł-FTT which includes part of $T^{Ev}$ and whose special axioms (provable formulas) are just those derived from the corresponding linguistic descriptions. Thus, only those formulas will be used that relate directly to the given situation (we may speak about *belief state* — cf. [2]). We will also consider a specific model $\mathcal{M}$ given by the evidence above.

#### 5.2.1 First derivation

**Conclusion from evidence (EvdC1) of Mr. Carry:** *Drive duration of Mr. Carry was big.*

PROOF: Let $\mathbf{d}_0$ be a constant representing the drive duration of Mr. Carry. Then $T \vdash \mathbf{d}_0 \equiv \top_w$ due to the given context. From this, we can prove that

$$T \vdash \mathit{Perc} \cdot \mathbf{d}_0 \,\mathrm{Int}(\mathit{Bi}_{very}). \tag{9}$$

From (9) we can further prove that $T \vdash \mathit{Perc} \cdot \mathbf{d}_0 \,\mathrm{Int}(\mathit{Bi})$ (by Lemma 2(a)) that is, $T \vdash \mathit{Eval} \cdot w_D \,\mathbf{d}_0 \,\mathrm{Int}(\mathit{Bi})$ and finally, with respect to the given context, $T \vdash \mathit{Bi}\, w_D \mathbf{d}_0$. □

**Lt. Columbo's conclusion:** *Drive duration of Mr. Carry is small, i.e. it is not big.*

Similarly as above, we formalize the evidence (Evd1) as follows: Let $\mathbf{t}_0$ be a constant representing temperature. With respect to the given context $w_T$ and the model $\mathcal{M}$, $\mathcal{I}_p^{\mathcal{M}}(\mathbf{t}^0) = 40$.

PROOF: The contraposition of (5) leads to the rule

$$\text{IF engine temperature is } \neg\, \mathit{Bi} \quad \text{THEN drive duration is } \neg\, \mathit{Bi} \tag{10}$$

Moreover, using (3) and Lemma 2(b), we conclude that

$$T \vdash \mathit{Perc} \cdot \mathbf{t}_0 \,\mathit{Sm}_\nu \tag{11}$$

for a suitable $\nu$ (this should correspond to the linguistic hedge "more or less") as well as

$$T \vdash (\forall x)(\exists y)\boldsymbol{\Delta}(\Upsilon(\mathit{Sm}_\nu\ w_T\, x) \Rightarrow \Upsilon(\neg\, \mathit{Bi}\ w_D\, y)). \tag{12}$$

Let us denote $\mathbf{b}^0 \equiv \mathit{Sm}_\nu\, w_T\, \mathbf{t}^0$ and denote the drive duration of Mr. Carry stemming from Lt. Columbo's reasoning by a constant $\mathbf{d}$. From (11) we get $T \vdash \Upsilon \mathbf{b}^0$. From this and (12), we obtain

$$T \vdash \mathbf{b}^0 \Rightarrow \neg\, \mathit{Bi}\, w_D\, \mathbf{d} \tag{13}$$

which implies $T \vdash \Upsilon(\neg\, \mathit{Bi}\, w_D\, \mathbf{d})$. The intention of Mr. Carry was to persuade Lt. Columbo that they speak about the same drive duration. From the formal point of view this provides a special axiom $T \vdash \mathbf{d}_0 \equiv \mathbf{d}$. But then we obtain

$$T \vdash \mathit{Bi}\, w_D\, \mathbf{d}_0 \,\&\, \Upsilon(\neg\, \mathit{Bi}\, w_D\, \mathbf{d}_0)$$

and using Lemma 2(d) we conclude that the theory $T$ is contradictory. Consequently, Mr. Carry lied.

□

#### 5.2.2 Second derivation

This derivation is here only outlined because of the lack of space.

**Lt. Columbo's conclusion:** *The wristwatch found on Mr. Smith hand does not belong to him.*

PROOF:

(L.1) IF state of wristwatch is $\mathit{Sm}$ THEN state of wristwatch is $\neg\, \mathit{Bi}$ (rule (3))

(L.2) IF state of wristwatch is $\neg\, \mathit{Bi}$ THEN strike is $\neg \mathit{ML}\, \mathit{Sm}$ OR quality of wristwatch is $\neg\, \mathit{Bi}$ (contraposition of (8), rules of FTT)

(L.3) IF Corr(R, S) is $\mathit{Sm}$ THEN Corr(R, S) is $\neg\, \mathit{Bi}$ (rule (3))

(L.4) IF Corr(quality of P's wristwatch, wealth of P) is $\neg\, \mathit{Bi}$ THEN $\neg$ (wristwatch belongs to P) (contraposition of (7), rules of FTT)

The evidence provided by Lt. Columbo leads to a sequence of perceptions. Furthermore, using perception-based logic deduction, we conclude that the *wristwatch does not belong to Mr. Smith.*

□

We see that Lt. Columbo's conclusion in Subsection 5.2.1 contradicts the evidence (EvdC1) given by Mr. Carry, hence Mr. Carry lies. We also see that Lt.. Columbo's conclusion in Subsection 5.2.2 contradicts the evidence (EvdC2) implicitly given by Mr. Carry, and we can conclude that he had an opportunity to kill Mr. Smith. Therefore, Lt. Columbo (and we) concluded that Mr. Carry assassinated Mr. Smith.

Let us stress that on many places, the reasoning described above is close to that in classical logic. This is correct since fuzzy logic generalizes but does not deny classical logic. The main point is, that we work in situations when evidence varies around the boundaries of vaguely-defined concepts. Fuzzy logic (and, namely, FTT) provides insight into the meaning of them and enables us to explain why and how we come from the vague content hidden inside to a more crisp surface form and make our conclusions.

## 6 Conclusion

In this paper, we demonstrated the power of fuzzy type theory on the example based on Lt. Columbo's case. We demonstrated that in Ł-FTT we can describe and solve non-trivial complex detective case.

In the further research, we will use a similar methodology in the analysis of economic situations. We also want to develop a translation procedure from natural language propositions to logical fuzzy IF-THEN rules and use automated proof techniques for derivation of conclusions.

## References


[1] P. Andrews. *An Introduction to Mathematical Logic and Type Theory: To Truth Through Proof.* Kluwer, Dordrecht, 2002.

[2] A. Bochman. *A Logical Theory of Nonmonotonic Inference and Belief Change.* Springer, Berlin, 2001.

[3] B. Diaz and T. Takagi. PNL applied to economics. In *Proc. Int. Conf. FUZZ-IEEE May 2005, Reno, USA*, 2005.

[4] A. Dvořák. *Fuzzy Logic Deduction in the Frame of Fuzzy Logic in Broader Sense.* PhD thesis, University of Ostrava, Ostrava, 2003.

[5] A. Dvořák and V. Novák. Fuzzy logic as a methodology for the treatment of vagueness. In L. Běhounek and M. Bílková, editors, *The Logica Yearbook 2004*, pages 141–151. Filosofia, Prague, 2005.

[6] V. Novák. Antonyms and linguistic quantifiers in fuzzy logic. *Fuzzy Sets and Systems*, 124:335–351, 2001.

[7] V. Novák. On fuzzy type theory. *Fuzzy Sets and Systems*, 149:235–273, 2005.

[8] V. Novák. Perception-based logical deduction. In B. Reusch, editor, *Computational Intelligence, Theory and Applications*, pages 237–250. Springer, Berlin, 2005.

[9] V. Novák. Fuzzy logic theory of evaluating expressions and comparative quantifiers. In *Proc. Int. Conf. IPMU, Paris, July 2006*, 2006.

[10] V. Novák. A comprehensive theory of evaluating linguistic expressions. In S. Gottwald and E. P. Klement, editors, *Proc. of 26th Linz Seminar on Fuzzy Set Theory.* Springer, Berlin, (to appear; (*Research Report* No. 71, http://irafm.osu.cz/irafm)).

[11] V. Novák and S. Lehmke. Logical structure of fuzzy IF-THEN rules. *Fuzzy Sets and Systems*, in press.

[12] V. Novák and I. Perfilieva. On the semantics of perception-based fuzzy logic deduction. *International Journal of Intelligent Systems*, 19:1007–1031, 2004.

[13] L.A. Zadeh. A note on web intelligence, world knowledge and fuzzy logic. *Data & Knowledge Engineering*, 50:291–304, 2004.

[14] L.A. Zadeh. Precisiated natural language. *AI Magazine*, 25:74–91, 2004.