University of Ostrava

Institute for Research and Applications of Fuzzy Modeling

Aggregation operators based fuzzy approximations

Martina Daňková, Susanne Saminger

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University of Ostrava
Institute for Research and Applications of Fuzzy Modeling
30. dubna 22, 701 03 Ostrava 1, Czech Republic

tel.: +420-59-6160234 fax: +420-59-6120 478
e-mail: Martina.Dankova@osu.cz

Abstract

In this contribution we enlarge the class of approximating techniques by those which use integral based aggregation operators.

Keywords: Fuzzy approximation, Aggregation operators.

1 Introduction

The paper contributes to the field of fuzzy approximation. Fuzzy approximation may be treated as a theory studying approximating functions created using techniques based on the theory of fuzzy sets. It is understood generally in two different ways differing by tool that we have at disposal intended to approximate a given function or a data-set: (1) dependency is expressed by fuzzy function or (2) it is approximated by an ordinary function using techniques based on fuzzy set theory. Both approaches have common feature which lies in transparent interpretability.

Later, we will focus only on the second type of fuzzy approximation techniques. The basic idea behind the approach introduced in this paper is to apply two times aggregation operator (similarly as proposed in [2] or later in [5]):

1'th aggregate data on subsets of input-space, and then

2'nd aggregate this local information into the global one.

The first part of this work may be treated as a generalization of approximation method for continuous functions (discrete data relating to continuous function) introduced by Perfilieva in [6] as the technique called fuzzy transformation. There, the weighted arithmetic mean is used only, while in this approach, approximations using Choquet-like aggregation operators (leading to weighted quasi-arithmetic means) are presented. This aggregation can be seen also as a transformation of the input data with the aim to obtain symmetrical spread around the final central characteristic. Using the Choquet-like aggregation operators, we can capture and eliminate different kinds of noise or an error included in the system etc.

The second part is devoted to approximations based on Sugeno-like integrals [9] and may be treated as an alternative to fuzzy transformation using operations on residual lattice. Unlike in [6], we follow the main idea of applying aggregating operator to local subsets by defining maxitive fuzzy measure on on these subsets and afterwards on the whole domain.

The main goal of this contribution is to investigate whether integral-based aggregation operators (weighted quasi-arithmetic means or weighted maximum etc.) can be used to approximate a given function with an arbitrary precision. Moreover, we hope to find approximating functions which minimize different criterions.

2 Preliminaries

Before turning to the main investigations let us recall some basic notions and concepts.

We assume a function f on a nonempty set X and denote by $X_n = \{\vec{x}_{(n)} = (x_1, \dots, x_n) | \, x_i \in X, i = 1, \dots, n\}$ the input-tuples for a fixed number of inputs and by $\mathbf{X}_n$ the corresponding index set $\{1, \dots, n\}$. Moreover, we consider $\mathcal{A}$ to be the σ-algebra of subsets of X_n, $(X_n, \mathcal{A})$ which is a measurable space. By $g : [0,1] \to [0,1]$ (or $g : [0,1] \to [0,\infty]$) we denote a continuous strictly monotone function which might be used for transforming input data. We say that a $\mathbf{m} : \mathcal{A} \to [0,1]$ is a fuzzy measure on X_n, if

(1) $\mathbf{m}(\emptyset) = 0$ and $\mathbf{m}(X_n) = 1$,

(2) $\mathbf{m}(I) \leq \mathbf{m}(J)$, whenever $I \subset J \subseteq X$.

It is moreover additive, if

(3) $\mathbf{m}(J) = \sum_{i \in J} w_i$, where $w_i = \mathbf{m}(\{i\})$, $i \in X_n$.

For the sake of brevity, we will write $gf(x)$ instead of $(g \circ f)(x)$ or $g(f(x))$ for arbitrary g, f.

Choquet-like integrals (introduced in [4]) for f are related to additive measures and can be expressed as integrals in the following form:

$$f_g^C(\vec{x}_{(n)}) = g^{-1}\left((C) - \int_{\mathbb{X}_n} gf \, dg\mathbf{m}\right) = g^{-1}\left[\sum_{i=1}^{n} gf(x_i)(g\mathbf{m}(I_i) - g\mathbf{m}(I_{i+1}))\right], \tag{1}$$

where $\vec{x}'_{(n)}$ is non-decreasing permutation of the input n-tuple $\vec{x}_{(n)}$ and $x'_0 = 0$, $x'_{n+1} = \infty$ by convention. Further, if (a(1),...,a(n)) is a permutation of indexes (1,...,n), such that $x'_i = x_{a(i)}$, then $I'_i = \{a(i), \ldots, a(n)\}$, i=1,...,n, and $I'_{n+1} = \emptyset$. As stated in [1], f_g^C is an idempotent continuous n-ary aggregation operator which is pseudo-linear and comonotone pseudo-additive with respect to the pseudo-addition $\oplus$ and pseudo-multiplication $\odot$, where

$$x \oplus y = g^{-1}(\min(g(1), g(x) + g(y))), \tag{2}$$

$$x \odot y = g^{-1}(g(x) \cdot g(y)), \tag{3}$$

for each $x, y \in [0, 1]$.

Note that if we fix the order of the input n-tuple $\vec{x}_{(n)}$ then (1) becomes a weighted quasi-arithmetic mean that is idempotent and bisymmetric aggregation operator (see also [1]).

Sugeno-like integrals are based on some $[0, 1]$-valued function f, some pseudo-multiplication $\odot$ and some fuzzy measure $\mathbf{m}$:

$$f_\odot^S(\vec{x}_{(n)}) = (S) - \int_{\mathbb{X}_n} f \, d\mathbf{m} = \bigvee_{i=1}^{n} f(x'_i) \odot \mathbf{m}(I'_i), \tag{4}$$

where $\vec{x}'_{(n)}$ is non-decreasing permutation of the input n-tuple $\vec{x}_{(n)}$ and $x'_0 = 0$, $x'_{n+1} = \infty$ by convention. Further, if $(a(1), \ldots, a(n))$ is a permutation of indexes $(1, \ldots, n)$, such that $x'_i = x_{a(i)}$, then $I'_i = \{a(i), \ldots, a(n)\}$, $i = 1, \ldots, n$, and $I'_{n+1} = \emptyset$. The integral is named after Sugeno ([9], 1974), where he introduced (4) with $\odot = \wedge$ and even before by Shilkret ([8], 1971) with $\odot = \cdot$. Note that instead of the pseudo-multiplication, we may use an arbitrary t-norm with no zero divisors as published by Weber ([10], 1986) such that (4) is refereed to as Sugeno-Weber integral. In general, we may assume arbitrary t-norm but in this case, (4) may appear to be zero for positive-constant function on set with positive measure. Remind that an arbitrary t-norm $*$ is distributive w.r.t. $\vee$ (Proposition 1.4.6. in [3]), i.e. $(x_1 \vee x_2) * y = (x_1 * y) \vee (x_2 * y)$, therefore, we still keep the important property of max-homogeneity for all $a \in [0, 1]$, i.e.

$$a \vee f_*^S(\vec{x}_{(n)}) = f_*^S(a \vee \vec{x}_{(n)}),$$

and also comonotone maxitivity, i.e.

$$f_*^S(\vec{x}_{(n)}) \vee f_*^S(\vec{y}_{(n)}) = f_*^S(\vec{x}_{(n)} \vee \vec{y}_{(n)}).$$

Note that all aggregation operators based on Sugeno-like integrals are continuous, idempotent aggregation operators which are comonotone maxitive.

3 Integral based approximation

In the following text, we will assume $X = [0, 1]$. Let $g : [0, 1] \to [0, 1]$ be a continuous strictly increasing bijection. We define a metric on X by

$$\varrho(x, y) = |g(x) - g(y)|,$$

where $g : [0, 1] \to [0, 1]$ is a continuous strictly increasing. On the basis ϱ, we can define the following metric

$$x \leftrightarrow_\varrho y = g^{-1}(|g(x) - g(y)|).$$

Remark 3.1 *For the simplicity we work with $X = [0, 1]$. But all the following results pass also for X such that $T(X) \subseteq [0, 1]$, where T is continuous strictly increasing.*

3.1 Choquet-like integral based approximation

In the sequel, we will fix fuzzy measures in order to specify aggregation operators which are used for the approximation, i.e., for the computation of the local estimation of a function f by basic functions and finally the approximation of f based on the local estimates. Let us remind that weighted quasi-arithmetic mean is a special case of the Choquet-like integral, and the later specification leads to that specific class of aggregation operators.

Let $\mathbb{A} = \{A_1, \ldots, A_k\}$, where each $A_i : X \to [0,1]$ is continuous, creates a Ruspini's-like fuzzy partition (modified version of Ruspini's partition from [7]), i.e. $\sum_{i=1}^{k} gA_i(x) = g1$ for each $x \in X$. Then, the operation

$$\mathbf{m}_x(\mathcal{X}) = g^{-1}\left(\sum_{A\in\mathcal{X}} gA(x)\right),$$

defines fuzzy measure of $\mathcal{X} \subseteq \mathbb{A}$ on $\mathbb{A}$ for each $x \in X$.

Let $D_n = \{x_i \in X \mid i = 1, \ldots, n\}$ be the data set. Then,

$$\mathbf{m}_i(\mathcal{X}) = g^{-1}\left(\sum_{x\in\mathcal{X}} \frac{gA_i(x)}{\sum_{y\in D_n} gA_i(y)}\right),$$

defines fuzzy measure of $\mathcal{X} \subseteq D_n$ on D_n for each $i = 1, \ldots, k$.

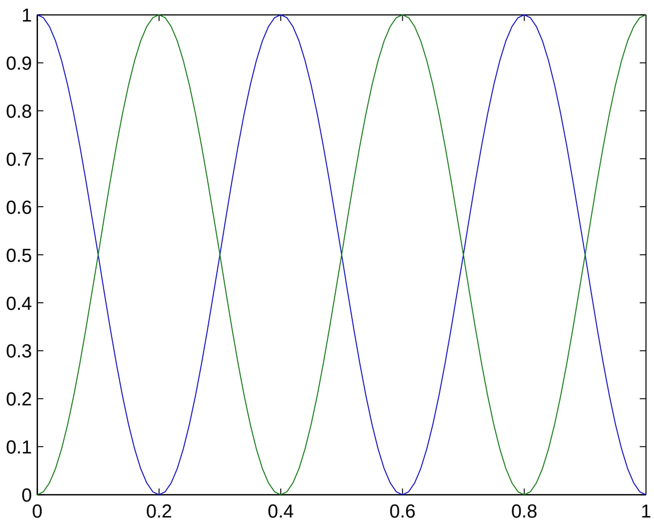

Figure 1: Example of Ruspini's partition

Definition 3.2 *Let $\mathbf{m}_1, \ldots, \mathbf{m}_k, \mathbf{m}_x$ be as above. Then a Choquet-like integral w.r.t. $\mathbf{m}_i$*

$$f_{i,g} = g^{-1}\left(\frac{1}{\sum_{k=1}^{n} gA_i(x_k)} \sum_{j=1}^{n} gA_i(x_j) \cdot gf(x_j)\right), \tag{5}$$

is weighted quasi-arithmetic mean w.r.t. $\mathbf{m}_i$ for each $i = 1, \ldots, k$. And

$$F_g(x) = g^{-1}\left(\sum_{i=1}^{k} gA_i(x) \cdot g(f_{i,g})\right), \tag{6}$$

is weighted quasi-arithmetic mean w.r.t. $\mathbf{m}_x$ for each $x \in X$, that we call Arithmetic-transformation of the data set $f(D_n) = \{f(x) \mid x \in D_n\}$.

For those, who are familiar with weighting triangles, we can say that $f_{i,g}$ and $F_g(x)$ are weighted quasi-arithmetic means w.r.t. weighting triangles $\triangle_i$ and $\triangle_x$, respectively, where

$$\triangle_x = (gA_i(x)|\ i = 1, \ldots, k) \quad \text{and}$$
$$\triangle_i = \left(\frac{gA_i(x_p)}{\sum_{j=1}^n gA_i(x_j)} \mid p = 1, \ldots, n \right).$$

Clearly, $f_{i,g}$ and $F_g(x)$ minimize variance of $f(\vec{x}_{(n)}) = (f(x_1), \ldots, f(x_n))$ w.r.t. weighting triangle $\triangle_i$ and variance of $f_{1,g}, \ldots, f_{k,g}$ w.r.t. $\triangle_x$ respectively.

Lemma 3.3 *Let $f_{i,g}$ and $F_g(x)$ be as above. Then,*

(1) $y = gf_{i,g}$ *minimizes* $\sum_{j=1}^n gA_i(x_j)(gf(x_j) - y)^2$,

(2) $y = gF_g(x)$ *minimizes* $\sum_{i=1}^k gA_i(x)(gf_{i,g} - y)^2$, *where* $x \in X$.

Proof:

(1) We need to solve the following equation $-2\sum_{j=1}^n gA_i(x_j)(gf(x_j)-y) = 0$, which leads to $\sum_{j=1}^n gA_i(x_j)gf(x_j) = y\sum_{j=1}^n gA_i(x_j)$, and finally

$$y = \frac{\sum_{j=1}^n gA_i(x_j)gf(x_j)}{\sum_{j=1}^n gA_i(x_j)} = gf_{i,g}.$$

(2) Analogously, $\sum_{i=1}^k gA_i(x)(gf_{i,g} - y) = 0$ and put $y = gz$ then

$$z = z \odot 1 = g^{-1}\left(y \sum_{i=1}^k gA_i(x) \right) = g^{-1}\left(\sum_{i=1}^k gA_i(x)gf_{i,g} \right) = F_g(x),$$

because $\forall x \in X : \sum_{i=1}^k gA_i(x) = 1$.

QED

Now we are going to show that using Arithmetic-transformation we are able to approximate a given data with an arbitrary precision.

Theorem 3.4 *Let f be a continuous function on the metric space (X, ϱ). Then for arbitrary $n \in \mathbb{N}$ and $\varepsilon > 0$ there exists $k \in \mathbb{N}$ and Ruspini's-like partition $\{A_1, \ldots, A_k\}$ such that F_g ε-approximates $f(D_n)$, i.e.*

$$|gf(x) - gF_g(x)| \le \varepsilon, \tag{7}$$

for each $x \in D_n$.

Proof: The proof is analogous to the one given in [6] for weighted arithmetic mean. Let us fix n. Since f is continuous then for arbitrary $\varepsilon > 0$ there exists $\delta > 0$ such that for all $x, y \in X$:

$$|g(x) - g(y)| \le \delta \Rightarrow |gf(x) - gf(y)| \le \varepsilon,$$

$i = 1, \ldots, n$. Now, let us fix some ε and find δ fulfilling the inequality above. Next, take $C = \{c_i = 2(i-1)\delta|\ i = 1, \ldots, k+1\}$, where $k = \frac{1}{2\delta}$ and define fuzzy partition $\{A_1, \ldots, A_{k+1}\}$, $k + 1 \le n$, such that each A_i is continuous on X, $A_i(c_i) = 1$ for all $i = 1, \ldots, k+1$ and each $gA_i(x) = 0$ for all $x \in X \setminus (c_{i-1}, c_{i+1})$, where $c_0 = -\frac{1}{2\delta}$ and $c_{k+2} = \frac{2\delta+1}{2\delta}$. Therefore, for each $x \in D_n$ there exists i such that $gA_i(x) > 0$, which implies $|g(x_j) - g(c_i)|, |g(x) - g(c_i)| < \delta/2$ and hence

$$|g(x_j) - g(x)| \le |g(x_j) - g(c_i)| + |g(x) - g(c_i)| < \delta,$$

by continuity we obtain that $|gf(x) - gf(x_j)| \le \varepsilon$ for each $j = 1, \ldots, n$.

The rest of the proof stands in verification of (11). Clearly

$$|gf_{i,g} - gf(x)| = \left| \frac{\sum_{j=1}^{n} gA_i(x_j)gf(x_j)}{\sum_{k=1}^{n} gA_i(x_k)} - gf(x) \right|$$
$$= \frac{\sum_{j=1}^{n} gA_i(x_j)|gf(x_j) - gf(x)|}{\sum_{k=1}^{n} gA_i(x_k)} \leq \frac{\sum_{j=1}^{n} gA_i(x_j) \cdot \varepsilon}{\sum_{k=1}^{n} gA_i(x_k)} = \varepsilon.$$

Which leads to

$$|gF_g(x) - gf(x)| = \left| \sum_{i=1}^{k} gA_i(x) \cdot gf_{i,g} - \sum_{i=1}^{k} gA_i(x)gf(x) \right| = \sum_{i=1}^{k} gA_i(x)|gf_{i,g} - gf(x)| \leq \varepsilon \sum_{i=1}^{k} gA_i(x).$$

Since g is strictly increasing, therefore we obtain

$$g^{-1}(|gF_g(x) - gf(x)|) \leq g^{-1}(\varepsilon) \odot g^{-1} \left(\sum_{i=1}^{k} gA_i(x) \right) = g^{-1}\varepsilon,$$

and hence $\forall x \in X : |gf_{i,g} - gf(x)| \leq \varepsilon$. QED

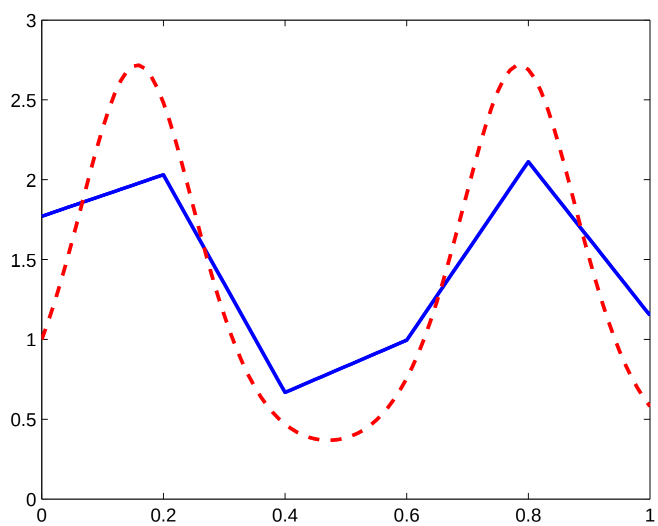

Figure 2: Weighted arithmetical mean approximation

Example 3.5 *Let $f(x) = \mathrm{e}^{\sin(10x)}$ on [0,1]. Perfilieva's F-transform constructed in 6 nodes gives result depicted on Figure 3.1. The red dashed line represents the original function f. Analogously in the case of approximation using $g = x^2$ with the same nodes which is depicted on Figure 3.1. Finally, comparison of the errors of approximations of f by F-transform (blue line) and g-approximation (red dashed line) is shown on Figure 3.1.*

We see that the g-approximation is better for the bigger values and the error grows with the decreasing functional values.

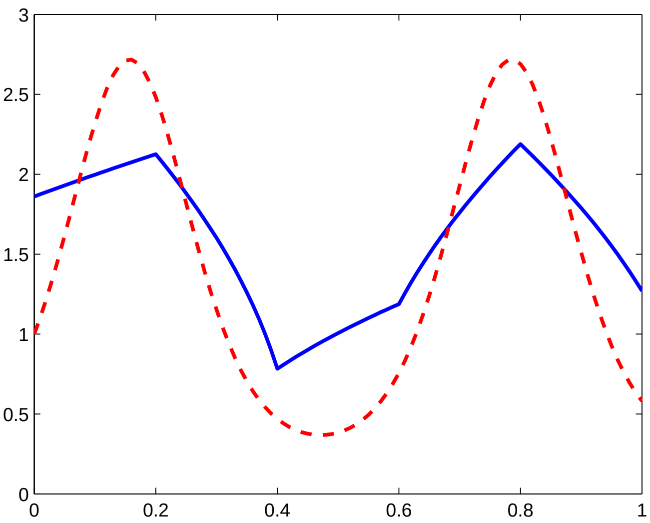

Figure 3: Approximation on the basis of $g = x^2$.

3.2 Sugeno-like integral based approximation

We will follow the approach from the previous section. Assume $*$ to be a strict t-norm, i.e. $x * y = g^{-1}(g(x) \cdot g(y))$, and $x \to y = \bigvee\{z \in [0,1] | \ x * z \leq y\}$ its residuum, then a structure

$$\mathcal{L} = \langle [0,1], *, \to, \wedge, \vee, 0, 1\rangle,$$

is a complete residuated lattice. Moreover, let us define an additional operation called biresiduum (or $*$-equivalence) by

$$x \leftrightarrow y = (x \to y) * (y \to x).$$

Notice that $\leftrightarrow_\varrho$ is different from $\leftrightarrow$.

Let $\mathbb{A} = \{A_1, \ldots, A_k\}$, where each $A_i : X \to [0,1]$ be continuous, creates a maxitive fuzzy partition, i.e. $\bigvee_{i=1}^{k} A_i(x) = 1$ for each $x \in X$. Therefore, the operation

$$\mathbf{m}_x(\mathcal{X}) = \bigvee_{A \in \mathcal{X}} A(x),$$

defines fuzzy measure of $\mathcal{X} \subseteq \mathbb{A}$ on $\mathbb{A}$ for each $x \in X$. Let $D_n = \{x_i \in X | \ i = 1, \ldots, n\}$ be the data set. Then,

$$\mathbf{m}_i(\mathcal{X}) = \bigvee_{y \in D_n} A_i(y) \to \bigvee_{x \in \mathcal{X}} A_i(x), \tag{8}$$

defines fuzzy measure of $\mathcal{X} \subseteq D_n$ on D_n for each $i = 1, \ldots, k$.

To find that (8) is the fuzzy measure, we need to realize that $a \to a = 1$.

Definition 3.6 *Let $\mathbf{m}_x, \mathbf{m}_1, \ldots, \mathbf{m}_k$ be as above. Then Sugeno-like integral w.r.t. $\mathbf{m}_i$ given by*

$$f_{i,*}(\vec{x}_{(n)}) = \bigvee_{k=1}^{n} A_i(x_k) \to \bigvee_{j=1}^{n} A_i(x_j) * f(x_j), \tag{9}$$

is the weighted quasi-maximum and we write $f_{i,*}$ w.r.t. A_i. And

$$F_*(x) = \bigvee_{i=1}^{k} A_i(x) * f_{i,*}, \tag{10}$$

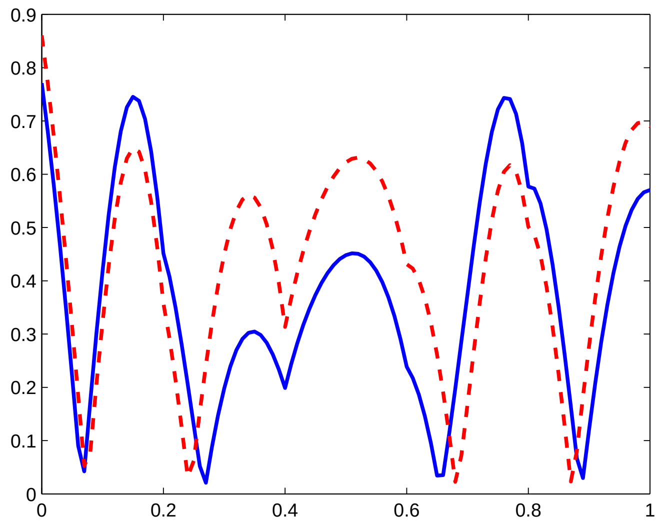

Figure 4: Comparison of the errors of approximations

is Sugeno-like integral w.r.t. $\mathbf{m}_x$ *for each* $x \in X$*, that we will called Max-transformation of the data set* $f(D_n) = \{f(x) | \, x \in D_n\}$.

Speaking freely, $f_{i,*}$ minimizes the variance w.r.t. $\{f(x_1), \ldots, f(x_n)\}$ relatively to the fuzzy set A_i and analogously, $F_*(x)$ minimizes the variance w.r.t. $\{f_{1,*}, \ldots, f_{k,*}\}$ for each $x \in X$.

Lemma 3.7 *Let* $f_{i,*}$*,* $F_*(x)$ *be as above,* $*$ *be a strict t-norm, and* $p = \bigvee_{k=1}^{n} A_i(x_k)$*. Then*

(1) $y = f_{i,*}$ *minimizes* $g\left(p \rightarrow \bigvee_{j=1}^{n} A_i(x_j) * (f(x_j) \leftrightarrow_\varrho y)^2\right)$,

(2) $y = F_*(x)$ *minimizes* $g\left(\bigvee_{i=1}^{k} A_i(x) * (f_{i,g} \leftrightarrow_\varrho y)^2\right)$, *where* $x \in X$.

Proof:

(1) We have to solve the following equation $\frac{1}{gp} \bigvee_{j=1}^{n} gA_i(x_j) \cdot (gf(x_j) - gy) = 0$, i.e. $\frac{1}{gp} \bigvee_{j=1}^{n} gA_i(x_j) \cdot gf(x_j) = \frac{1}{gp} \bigvee_{j=1}^{n} gA_i(x_j) \cdot gy = gy$.

(2) Analogously, we solve $\bigvee_{i=1}^{k} gA_i(x) \cdot (gf_{i,*} - gy) = 0$, from which it immediately follows $\bigvee_{i=1}^{k} gA_i(x) \cdot gf_{i,*} = gy \cdot \bigvee_{i=1}^{k} gA_i(x) = gy$ for each $x \in X$.

QED

Now we are going to show that using Max-transformation we are able to approximate a given data with an arbitrary precision.

Theorem 3.8 *Let* f *be a continuous function on a metric space* (X, ϱ)*. Then for arbitrary* $n \in \mathbb{N}$ *and* $\varepsilon > 0$ *there exists* $k \in \mathbb{N}$ *and maxitive partition* $\{A_1, \ldots, A_k\}$ *such that* F_* ε*-approximates* $f(D_n)$*, i.e.*

$$\varrho(f(x), F_*(x)) \leq \varepsilon, \tag{11}$$

for each $x \in D_n$.

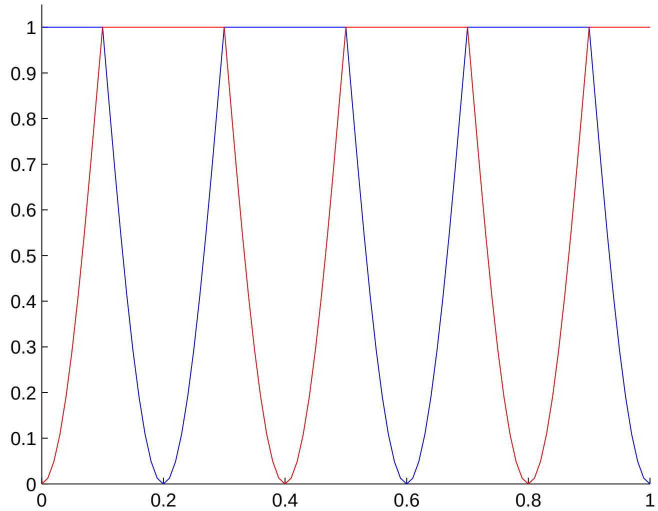

Figure 5: Example of maxitive partition

Proof: Let us fix n. Since f is continuous then for arbitrary $\varepsilon > 0$ there exists $\delta > 0$ such that for all $x, y \in X$: $\varrho(x,y) \leq \delta$ implies $\varrho(f(x), f(y)) \leq \varepsilon$, $i = 1, \dots, n$. Now, let us fix some ε and find δ fulfilling the inequality above. Next, take $C = \{c_i = 2(i-1)\delta |\ i = 1, \dots, k+1\}$, where $k = \frac{1}{2\delta}$ and define fuzzy partition $\{A_1, \dots, A_k\}$, $k+1 \leq n$, such that each A_i is continuous on X and

$$A_i(x) = \begin{cases} 1, & x \in [c_i, c_{i+1}], \\ 0, & x \in [c_{i-1}, c_{i+2}]. \end{cases}$$

Therefore, for each $x \in D_n$ there exists i such that $A_i(x) > 0$, which implies $\varrho(x_j, c_i), \varrho(x, c_i) < \delta$ and hence

$$\varrho(x_j, x) \leq \varrho(x_j, c_i) \vee \varrho(x, c_i) < \delta,$$

by continuity we obtain that $\varrho(f(x), f(x_j) \leq \varepsilon$ for each $j = 1, \dots, n$.

The rest of the proof stands in verification of (11). Let $p = \bigvee_{k=1}^{n} A_i(x_k)$ then we can write

$$\begin{aligned} |gf_{i,g} - gf(x)| &= \\ &= \left| \frac{1}{gp} \bigvee_{j=1}^{n} \underbrace{gA_i(x_j) \cdot gf(x_j)}_{q_i} - \frac{gp}{gp}\, gf(x) \right| = \left| \frac{1}{gp} \bigvee_{j=1}^{n} q_i - \frac{1}{gp} \bigvee_{j=1}^{n} gA_i(x_j) \cdot gf(x) \right| \\ &= \frac{1}{gp} \bigvee_{j=1}^{n} gA_i(x_j) \cdot |gf(x_j) - gf(x)| \leq \frac{1}{gp} \bigvee_{j=1}^{n} gA_i(x_j) \cdot \varepsilon = \varepsilon. \end{aligned}$$

This finally leads to

$$\begin{aligned} |gF_g(x) - gf(x)| &= \left| \bigvee_{i=1}^{k} gA_i(x) \cdot gf_{i,g} - \bigvee_{i=1}^{k} gA_i(x) \cdot gf(x) \right| \\ &= \bigvee_{i=1}^{k} gA_i(x) \cdot |gf_{i,g} - gf(x)| \leq \varepsilon \cdot \bigvee_{i=1}^{k} gA_i(x) = \varepsilon. \end{aligned}$$

QED

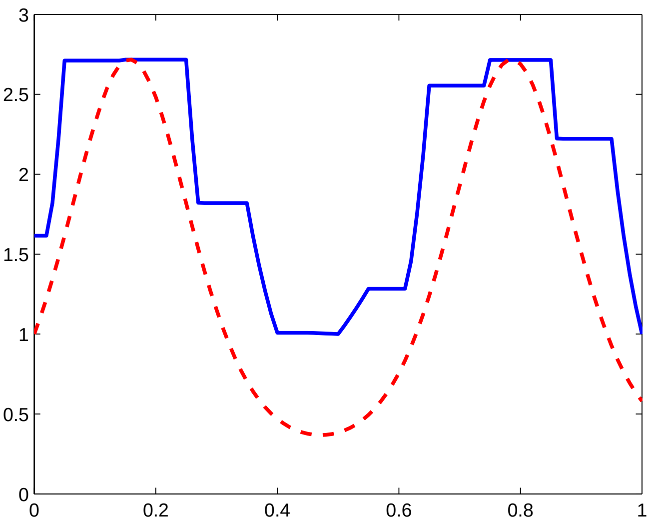

Figure 6: Weighted maxitive approximation

Remark 3.9 *The same results can be easily received for nilpotent t-norm as well as for minimum whenever we assume maxitive partition such that* $\bigvee_{j=1}^{n} A_i(x_j) = 1$ *for each* $i = 1, \dots, k$ *and metric dual to* $\leftrightarrow$.

Example 3.10 *Assume the same* f *as in Example 3.5. Figure 6 shows the result of approximating using 10 fuzzy intervals and the ordinary product.*

4 Conclusions

The class of aggregation operators is very broad and hence the number of possibilities how to create an approximating function is unlimited. Important is to create such approximation that fulfills required criterion. In this work, we have minimized the weighted variances associated to the given aggregation operators. The approximation in the case of Choquet-like integrals gives satisfactory results, while the Sugeno-like integrals does not look so promising. This is caused by the degrees of freedom relating to maxitive partition of a input domain. Therefore, the choice of domain partition is crucial and deserves a deeper study.

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