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Approximation Based Fuzzy Control

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Abstract

A new fundamental idea of expressing expert knowledge in a form fuzzy rules base (FRB for short) has been brought to the control theory. However, for some processes an expert knowledge acquisition is not a trivial task although there exist experts able to control such processes. In that cases, a data-driven approach of a construction of an FRB is used. This sometimes leads to such control techniques which in fact use fragments of the fuzzy set theory but on the other hand they controvert the incompatibility principle which was surely one of the main Zadeh's motivation for his inventions.

1 Introduction

Since 1960's a new methodology has been brought to the control theory by L.A. Zadeh who introduced a fundamental idea of expressing dependencies between variables by conditional sentences called fuzzy rules. An expert knowledge is then represented in a form of so-called *fuzzy rule base* (FRB).

However, for some systems an expert knowledge acquisition is not a trivial task or transformation of such knowledge into an FRB is technically hardly feasible [1]. For these cases a data-driven approach has to be used either to adapt some initial FRB or to generate a new one if no initial FRB is attainable. In general, data-driven approaches (neural learning, heuristic algorithms, adaptation optimizing a cost function etc.) deal with some training data obtained by experiments.

In the case of a generation of an FRB we just approximate given data but it might be done by any classical approach and thus the usage of *fuzzy* is questionable although it surely provides some advantages e.g. interpretability, transparency or robustness. In the second case we assume that we are given some initial FBR which is to be adapted. But an adaptation algorithm can lead to something completely different from the initial FRB and therefore it might be set randomly. So, we either do not have an expert knowledge or we lose it.

2 Fuzzy Transform

We propose a data-driven approach based on a fuzzy approximation method called fuzzy transform [15] which was proposed by I. Perfilieva; see [18]. This technique deals with antecedent fuzzy sets $\mathbf{A}_1, \ldots, \mathbf{A}_n$ (called *basis functions*) determining a fuzzy partition [20] of a given domain $X = [a, b]$. There are reals F_i in the respective consequents given by

$$F_i = \frac{\int_a^b \mathbf{A}_i(x) f(x) dx}{\int_a^b \mathbf{A}_i(x) dx} \tag{1}$$

where $f : X \to Y \subset \mathbb{R}$ is the approximated function. The interpretation is given by

$$f_n^F(x) = \sum_{i=1}^{n} \mathbf{A}_i(x) F_i. \tag{2}$$

Formulas (1) and (2) are called the fuzzy transform (the F-transform for short) and the inverse fuzzy transform (the inverse F-transform for short), respectively. Besides many useful properties it has been shown that the inverse F-transform uniformly converges to the original approximated function f; see [14, 15, 18]. For an extension for functions with more variables we refer to [22, 23].

If we have only discrete knowledge of the approximated function f i.e. we are given some measured (training) data $(x_j, f(x_j))$ $j = 1, \ldots, k$ then the definite integrals in formula (1) are replaced by sums i.e.

$$F_i = \frac{\sum_{j=1}^{k} \mathbf{A}_i(x_j) f(x_j)}{\sum_{j=1}^{k} \mathbf{A}_i(x_j)}. \tag{3}$$

The technique of the F-transform has been already successfully applied to many problems e.g. differential equations [14], noise removing [19], partial differential equations [23, 25] or data compression [16].

3 Extension for Fuzzy Relations

In fuzzy control, a control function f is usually replaced by a fuzzy relation $\mathbf{F}$ which is an interpretation of some FRB describing the controlled process. The F-transform technique can be easily extended for an arbitrary continuous fuzzy relation $\mathbf{F} : X \times Y \to [0,1]$; see [21]. If we are given a training data $(x_j, f(x_j))$ where $f(x_j)$ are control actions made by an expert during experiments then these control actions are fuzzified to get a data set $(x_j, \mathbf{F}(x_j, \cdot))$, where $\mathbf{F}(x_j, \cdot) \underset{\sim}{\subset} Y$. It leads to the following modification of formula (3)

$$\mathbf{F}_i(y) = \frac{\sum_{j=1}^{k} \mathbf{A}_i(x_j)\mathbf{F}(x_j, y)}{\sum_{j=1}^{k} \mathbf{A}_i(x_j)}. \tag{4}$$

Formula for the inverse F-transform of a fuzzy relation is analogous to (2) :

$$\mathbf{F}_n^F(x, y) = \sum_{i=1}^{n} \mathbf{A}_i(x)\mathbf{F}_i(y). \tag{5}$$

For a better understanding, it can be viewed as an FRB consisting of n fuzzy rules

$$\text{IF } x \text{ IS } \mathcal{A}_i \text{ THEN } y \text{ IS } \mathcal{F}_i \tag{6}$$

where $\mathcal{A}_i$ and $\mathcal{F}_i$ are linguistic expressions represented by fuzzy sets $\mathbf{A}_i$ and $\mathbf{F}_i$, respectively.

An interpretation of such FRB is done by formula (5). Since the basis functions $\mathbf{A}_i$ defined by I. Perfilieva [15, 18] fulfil the Ruspini condition [20] i.e. $\sum_{i=1}^{n} \mathbf{A}_i(x) = 1$ for all $x \in X$, formula (5) can be rewritten as follows

$$\mathbf{F}_n^F(x, y) = \bigoplus_{i=1}^{n} (\mathbf{A}_i(x) \odot \mathbf{F}_i(y)) \tag{7}$$

where $\oplus$ is the Łukasiewicz t-conorm and $\odot$ is the product t-norm. Such interpretation given by the right hand side of (7) will be called the *additive interpretation* of an FRB.

4 Approximation Properties

Let us mention some properties of the introduced additive interpretation of an FRB. Its original roots are hidden in so called additive normal form introduced in [17] and generalized in [5, 6].

Definition 1 Let $\mathbf{R}$ be a fuzzy relation on $X \times Y$ and $f : X \to Y$. We say that the relation $\mathbf{R}$ ε-approximates the function f if

$$\forall x \in X \forall y \in Y : \quad \mathbf{R}(x, y) \geq 0 \Rightarrow \mid y - f(x) \mid < \varepsilon.$$

From many publications investigating approxiation abilities of FRB systems we know that there exists an FRB system with the so called *disjunctive interpretation* (maximum of conjunctions) which ε-approximates an arbitrary continuous function on a compact domain; see [3, 4, 9, 13]. Here we claim that the additive interpretation keeps the same property.

Proposition 1 *Let X, Y be real closed intervals and let $f : X \to Y$ be an arbitrary continuous function. Then there exists an FRB system with the additive interpretation (7) which ε-approximates f for any $\varepsilon > 0$.*

PROOF: The proof is a slight modification of that one published in [13]. Since f is conitnuous on X

$$\forall \varepsilon \; \exists \delta : \; \forall x, x' \in X \;\; \mid x - x' \mid < \delta \Rightarrow \mid f(x) - f(x') \mid < \varepsilon/2$$

Put $h = \frac{|X|}{n-1} < \delta$ and $x_1 = \inf\{x \mid x \in X\}$, $x_i = x_{i-1} + h$ for $i = 2, \ldots, n$.

Let us construct antecedent fuzzy sets $\mathbf{A}_i$: $\mathbf{A}_i(x_i) = 1$; $\mathbf{A}_i(x) > 0$ if and only if $x \in (x_{i-1}, x_{i+1})$ where $x_0 = x_1, x_{n+1} = x_n$.

Denote the support of $\mathbf{A}_i$ by symbol U_i. Then $f(U_i)$ is an interval on Y. Denote the center of $f(U_i)$ by y_i and $V_i = (y_i - \varepsilon/2, y_i + \varepsilon/2)$. From the continuity of f we know that $\mid f(U_i) \mid < \varepsilon/2$ and therefore $f(U_i) \subset V_i$.

Let us construct arbitrary consequent fuzzy sets $\mathbf{F}_i$ such that $\mathbf{F}_i(y) > 0$ if and only if $y \in V_i$. Take an arbitrary $x' \in X$ then either $\exists U_i, U_{i+1} : x' \in U_i$ and $x' \in U_{i+1}$ or $x' \in \{x_i \mid i = 1, \dots, n\}$. In the first case $\mathbf{A}_i(x') > 0$ as well as $\mathbf{A}_{i+1}(x') > 0$ while $\mathbf{A}_j(x') = 0$ for $j \notin \{i, i+1\}$. Moreover we know that $f(x') \in V_i, V_{i+1}$.

If $\bigoplus_{i=1}^{n}(\mathbf{A}_i(x') \odot \mathbf{F}_i(y)) > 0$ then $\mathbf{A}_i(x')\mathbf{F}_i(y) + \mathbf{A}_{i+1}(x')\mathbf{F}_{i+1}(y) > 0$ and this occurs if and only if $\mathbf{F}_i(y) > 0$ or $\mathbf{F}_{i+1}(y) > 0$. Without loss of generality let $\mathbf{F}_i(y) > 0$ then $y \in V_i$.

Finally $\mid y - f(x') \mid = \mid y - y_i + y_i - f(x') \mid \leq \mid y - y_i \mid + \mid y_i - f(x') \mid \leq \varepsilon$.

The second case when $x \in \{x_i \mid i = 1, \dots, n\}$ is analogous. □

Definition 2 Let X be an arbitrary closed interval on $\mathbb{R}$ and $\mathbf{A}_i$, $i = 1, \dots, n$ are fuzzy sets on X. We say that $\mathbf{A}_i$ fulfil the orthogonality condition if

$$\bigoplus_{\substack{i=1 \\ i \neq j}}^{n} \mathbf{A}_i(x) = 1 - \mathbf{A}_j(x) \tag{8}$$

for all $x \in X$ and $j \in \{1, \dots, n\}$.

Remark 1 *By a direct computation one can easily check that the Ruspini condition implies orthogonality condition.*

Let us recall a proposition published in [5, 6] with a proof which directly uses a technique from [17]. The proposition clerifies a mutual position between the additive interpretation and standart interpretations.

Proposition 2 *Let an FRB system is described by n rules (6). Moreover, let the orthogonality condition (8) is fulfilled. Then the following inequalities hold*

$$\bigvee_{i=1}^{n}(\mathbf{A}_i(x) * \mathbf{F}_i(y)) \leq \bigoplus_{i=1}^{n}(\mathbf{A}_i(x) * \mathbf{F}_i(y)) \leq \bigwedge_{i=1}^{n}(\mathbf{A}_i(x) \rightarrow_{Ł} \mathbf{F}_i(y)), \tag{9}$$

where $\rightarrow_{Ł}$ is the Łukasiewicz implication and $*$ is an arbitrary left-continuous t-norm.

From the previous proposition it is obvious that the additive interpretation can be used with an arbitrary left continuous t-norm and the product t-norm is just a concrete case desired by the F-transform extension.

This section justifies our point of view to the F-transform as to an FRB system.

5 Additional Expert Knowledge

In any type of learning, there is a problem that the system must *learn* all possible situation since otherwise the system will not be able to behave correctly. This can lead to a huge mass of experiments and even this need not be sufficient.

Fuzzy transform can be helpful in this problem. After a reasonable number of experiments, an FRB with the consequent fuzzy sets given by (4) and the interpretation given by (7) is constructed. Such an FRB approximates the behaviour of an expert controlling the system during experiments.

In general, for an expert it is much easier to test an automatically generated FRB system and specify his knowledge for those situation which were not learned sufficiently than to build the whole FRB. Thereofore the FRB generated as above is tested and its behaviour observed. An expert proposes control actions for situations x_j, $j = k+1, \dots, k+r$ where the FRB does not work properly. These expert control actions are given linguistically i.e. by appropriate evaluating linguistic expression [11] which are represented by fuzzy sets $\mathbf{F}(x_j, \cdot) \subsetneq Y$.

Then the new consequent fuzzy sets $\mathbf{F}_i$ are computed again as the components of the extended F-transform i.e.

$$\mathbf{F}_i(y) = \frac{\sum_{j=1}^{k+r} \mathbf{A}_i(x_j)\mathbf{F}(x_j, y)}{\sum_{j=1}^{k+r} \mathbf{A}_i(x_j)}. \tag{10}$$

The F-transform components then aggregate both types of information - experimental one and expert one - to get a well tuned FRB system. It means that the data set is enriched by the expert knowledge and the original FRB generated by the F-transform is modified (recomputed).

6 Discussion

We recalled the fuzzy transform as an appropriate fuzzy approximation method for control tasks in this paper. Its natural extension for fuzzy relation was described as well.

The main aim of this extension is that the technique is “shape dependent” compared to the original one and allows a user to deal with a training data as well as with an expert knowledge in a linguistic form. As such, it provides a possibility to aggregate both information into one FRB.

Briefly, all fuzzy control techniques can be classified into to categories - the ones tackling expert knowledge (class 1) and the other ones which are more or less approximation based (class 2). The latter ones are used for a construction of FRB systems with a data-driven approach and with no linguistically given expert knowledge. This, according to Dubois, Prade and Ughetto [7], sometimes leads to such FBR systems which are more and more considered as standard non-fuzzy universal approximators. And such understanding is very surprising having in mind that the incompatibility principle was Zadeh’s motivation for introducing the concept of fuzzy sets [26]. On the other hand, many arguments for data-driven approaches have been provided; see [1].

To use advantages of both approaches some attempts have been done. Typically, some initial expert knowledge is available (class 1) and then the FRB is changed by some adaptation algorithm (class 2). This approach (class 1-2) has some drawbacks: again the problem with an availability of the initial FRB; the adaptation can completely change the initial FRB and therefore the expert knowledge is not presented anymore in the FRB (class 2).

To construct the FRB in the opposite way is a natural idea. The fact, that even purely linguistic models (class 1) like *perception based logical deduction* [10, 12] can be linked to some learning algorithm [2, 8] (class 2) providing some initial FRB gives a chance of a construction of balanced FRB systems (class 2-1). On the other hand, a system described in the previous lines has its drawbacks as well: the number of rules and therefore also the complexity and the accuracy is not in user’s hands; any additionally specified expert knowledge brings new rules what even increases the complexity and moreover, it can lead either to conflict rules or to redundant rules.

Our approach proposed in this paper allows to automatically generate an approximation based initial FRB (class 2) based on the F-transform (see [14, 15]) with all its advantages: complexity and accuracy in user’s (expert’s) hands. Moreover, it allows an expert to linguistically specify his knowledge (class 1) for some situations and add it to the FRB without any influence to the complexity or the redundancy and not cause a conflict of rules. The proposed method aggregates both sources of information - experimental and expert - into one FRB. As all method, it is not without drawbacks: although the method deals with an expert knowledge it is involved inside the FRB so, the FRB has no linguistic meaning - black box; the method is proposed just for one-level systems (no hierarchical systems) and so-called curse of dimensionality can cause computational problems.

In general, the technique is not claimed to be better than existing approaches but just tries to put both classes of FRB systems closer to each other and surely, besides mentioned disadvantages, brings also some advantages. As shown in [24], the approach was already successfully applied to a dynamic robot control. Since the approach avoids huge mass of experiments and a rough model is afterwards improved by an expert, an extremely low time requirements for the whole construction of the FRB was found out to be the most significant feature.

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