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Dynamic Robot Control Based on Fuzzy Approximation

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Abstract

Fuzzy approach to a process control is widely used for many advantages including the ability to deal with a linguistically given expert knowledge. However, because of apparent reasons, techniques for an automatic generation or adaptation of fuzzy rule bases from training data have been developed. In this paper, we deal with a control of a dynamic robot passing a corridor while having only partial information about his position. For this purpose we apply the fuzzy approximation approach which allows us to combine the automatic generation of rules from data with the expert knowledge.

K e y w o r d s: fuzzy transform; fuzzy approximation; fuzzy partition; fuzzy rule base; learning
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1 INTRODUCTION

A new methodology has been brought to the control theory by L.A. Zadeh who introduced a fundamental idea of expressing dependencies between variables by conditional sentences called fuzzy rules. This made possible to use a fragment of human language in control algorithms. An expert knowledge is represented in a form of fuzzy rules together forming a fuzzy rule base (FRB). Such systems using fuzzy rule bases are called fuzzy rule based systems and their main advantages are high robustness, capabilities to tackle uncertainty and vagueness and finally, their linguistic roots.

However, for some systems an expert knowledge acquisition is not a trivial task or transformation of such a knowledge into an FRB is technically hardly feasible (see [1]). For these cases, and not only for them, algorithms called *learning* are taken into account. The notion 'learning' is not necessarily in neural networks meaning, sometimes it is just an automatic generation of an FRB. Learning algorithms deal with a training data obtained by experiments. The training data serves us as a pattern of behaviour of a modelled system and the chosen learning algorithm transforms them into an FRB.

In this paper, we deal with a dynamic robot with a pivoted ultrasound sensor providing partial information about robot's position in a corridor. Robot's task is to pass through any chosen corridor. The given task is vague and any effort to obtain a precise mathematical formulation of the task leads either to something what is far from original human understanding of driving vehicles or to technically difficult systems like e.g. systems of partial differential equations etc. Therefore, we consider this problem to be perfect for an implementation of an FRB system.

This paper focuses on the application of the fuzzy transform (the *F-transform* for short) introduced in [3] and developed in [5] to the problem mentioned above. Such an approximation technique can be extended (see [10]) for fuzzy relations - a crucial mathematical notion behind FRB systems. By applying this technique to a given training data obtained by measuring driver's control actions we come to the learning algorithm which consists in approximating these control actions.

2 PRELIMINARIES

In this section, we recall basic definitions and ideas from [3, 4, 5] about the F-transform technique. This technique belongs to a new area called numerical methods on the basis of fuzzy approximation models.

Definition 1 Let $x_i = a + h(i-1)$ be nodes on $[a, b]$ where $h = (b-a)/(n-1)$, $n \geq 2$ and $i = 1, \ldots, n$. We say that functions $\mathrm{A}_1(x), \ldots, \mathrm{A}_n(x)$ defined on $[a, b]$ are *basis functions* if each of them fulfils the following conditions:

- $\mathrm{A}_i : [a, b] \to [0, 1]$, $\mathrm{A}_i(x_i) = 1$,
- $\mathrm{A}_i(x) = 0$ if $x \notin (x_{i-1}, x_{i+1})$, where $x_0 = a$, $x_{n+1} = b$,
- $\mathrm{A}_i(x)$ is continuous,
- $A_i(x)$ strictly increases on $[x_{i-1}, x_i]$ and strictly decreases on $[x_i, x_{i+1}]$,
- $\sum_{i=1}^{n} \mathrm{A}_i(x) = 1$, for all $x \in [a, b]$,

- $\mathrm{A}_i(x_i - x) = \mathrm{A}_i(x_i + x)$, for all $x \in [0, h]$, $i = 2, \dots, n-1$, $n > 2$,
- $\mathrm{A}_{i+1}(x) = \mathrm{A}_i(x - h)$, for all $x \in [a + h, b]$, $i = 2, \dots, n-2$, $n > 2$.

It is easy to see, that the basis functions form a fuzzy partition (see [8]) of the given domain and each basis function A_i can be viewed as a fuzzy number 'approximately x_i'.

Definition 2 Let f be a continuous function on $[a, b]$ and $\mathrm{A}_1, \dots, \mathrm{A}_n$ be basis functions. We say that the n-tuple of real numbers $[F_1, \dots, F_n]$ is the *F-transform* of f w.r.t. $\mathrm{A}_1, \dots, \mathrm{A}_n$ if

$$F_i = \frac{\int_a^b f(x)\mathrm{A}_i(x)dx}{\int_a^b \mathrm{A}_i(x)dx}. \tag{1}$$

Not always the function f is given analytically and for such in practice very usual cases, where only measurements $(x_p, f(x_p))$ $p = 1, \dots, q$ are provided, we define the F-transform as follows.

Definition 3 Let us be given data $(x_p, f(x_p))$ $p = 1, \dots, q$ on $[a, b] \times \mathbb{R}$ and $\mathrm{A}_1, \dots, \mathrm{A}_n$ be basis functions. We say that the n-tuple of real numbers $[F_1, \dots, F_n]$ is the *F-transform* of f w.r.t. $\mathrm{A}_1, \dots, \mathrm{A}_n$ if

$$F_i = \frac{\sum_{p=1}^q f(x_p)\mathrm{A}_i(x_p)}{\sum_{p=1}^q \mathrm{A}_i(x_p)}. \tag{2}$$

In both cases, the *components* F_i of the F-transform aggregate all the known information, either analytical or discrete, above non-zero supports of A_i. Then the vector of the F-transform serves us as a simplified discrete representation of f and can be successfully applied in complex computations (see [3, 11]).

The discrete representation is returned back by to the space of continuous functions to obtain a continuous approximation of function f.

Definition 4 Let $[F_1, \dots, F_n]$ be the F-transform of a function f w.r.t. $\mathrm{A}_1, \dots, \mathrm{A}_n$. The function

$$f_n^F(x) = \sum_{i=1}^n F_i A_i(x) \tag{3}$$

will be called the *inverse F-transform*.

The generalization for function with more variables is straightforward e.g. for two variables we construct fuzzy partitions on both axes with help of basis functions $A_i(x_1), B_j(x_2)$ where $i = 1, \dots, n$ and $j = 1, \dots, m$ and compute the components of the F-transform F_{ij}. For details see [11, 12]

The F-transform has been many times shown to be appropriate for a number of applications. Let us stress its main advantages and properties: linearity and uniform convergence [3, 4, 5], computational simplicity and fastness [3, 11], smoothing abilities [7, 11, 12], noise removing abilities [7], best approximation in integral sense [3, 4, 5].

3 F-TRANSFORM OF FUZZY RELATION

Fuzzy relation is a crucial mathematical notion behind the FRB systems. Instead of some crisp control function $f : X \to Y$ we deal with a fuzzy relation $\mathrm{R} : X \times Y \to [0, 1]$ which can be viewed as a fuzzy set-valued function $\mathrm{R} : X \to [0, 1]^Y$ i.e. as a mapping which assigns a fuzzy subset of Y to each node $x \in X$.

The original ideas of the F-transform from [3, 4, 5] can be easily extended for fuzzy relations (see [10]), which leads to the following formulas for the F-transform and its inversion

$$\mathrm{F}_i(y) = \frac{\int_a^b \mathrm{R}(x, y)\mathrm{A}_i(x)dx}{\int_a^b \mathrm{A}_i(x)dx}, \tag{4}$$

$$\mathrm{R}_n^F(x, y) = \sum_{i=1}^n \mathrm{F}_i(y)A_i(x), \tag{5}$$

respectively.

If we consider a data-based model i.e. R as a fuzzy set-valued function is known only at some nodes on X then the formula for the F-transform is again analogous to (2). So, let us be given data $(x_p, R(x_p, y))$ $p = 1, \ldots, q$, then the F-transform of R is given as follows

$$\mathrm{F}_i(y) = \frac{\sum_{p=1}^{q} \mathrm{R}(x_p, y)\mathrm{A}_i(x_p)}{\sum_{p=1}^{q} \mathrm{A}_i(x_p)}. \tag{6}$$

The form of the data is natural and can be obtain by asking an expert what to do when the input variable has value x_p and the expert answers imprecisely e.g. turn a wheel to the right for 'about y_p' degrees which leads to a fuzzy number $\mathrm{Y}_p(y) = \mathrm{R}(x_p, y)$.

The F-transform technique deals just with fuzzy relation and does not explicitly interpret an FRB but for better understanding we can imagine such an FRB composed by the following rules:

$$\text{IF } x \text{ IS } \mathcal{A}_i \text{ THEN } y \text{ IS } \mathcal{F}_i, \tag{7}$$

where $i = 1, \ldots, n$ and $\mathcal{A}_i$, $\mathcal{F}_i$ are linguistic evaluating expressions (see [2]) represented by the fuzzy sets A_i and F_i, respectively. Interpretation of this FRB is given by (5) what can be rewritten into the following form:

$$\bigoplus_{i=1}^{n} (\mathrm{A}_i(x) \odot \mathrm{F}_i(y)), \tag{8}$$

where $\oplus$ is the Łukasiewicz t-conorm and $\odot$ is the product t-norm. Indeed, since $\sum_{i=1}^{k} \mathrm{A}_i(x) = 1$ for all $x \in [a, b]$ and $\mathrm{F}_i(y) \leq 1$ we really obtain that

$$R_n^F(x, y) = \bigoplus_{i=1}^{n} (\mathrm{A}_i(x) \odot \mathrm{F}_i(y)).$$

The following proposition, which directly uses the result from [6] about disjunctive, conjunctive and additive normal forms and their mutual position, justifies our point of view to the F-transform as to an FRB system.

Proposition 1 *Let an FRB system is described by (7) where $i = 1, \ldots, n$ and $\mathcal{A}_i$, $\mathcal{F}_i$ are linguistic evaluating expressions represented by fuzzy sets A_i and F_i, respectively. Moreover, let the following generalized orthogonality condition is fulfilled*

$$\bigoplus_{\substack{i=1 \\ i=j}}^{n} \mathrm{A}_i(x) = 1 - \mathrm{A}_j(x) \quad j = 1, \ldots, n. \tag{9}$$

Then the following inequalities hold

$$\bigvee_{i=1}^{n} (\mathrm{A}_i(x) * \mathrm{F}_i(y)) \leq \bigoplus_{i=1}^{n} (\mathrm{A}_i(x) * \mathrm{F}_i(y)) \leq \bigwedge_{i=1}^{n} (\mathrm{A}_i(x) \rightarrow_{\L} \mathrm{F}_i(y)), \tag{10}$$

where $\rightarrow_{\L}$ is the Łukasiewicz implication and or and $*$ is an arbitrary left-continuous t-norm.

The technique of the proof can be found in [6]. The basis functions given by Definition 1 obviously fulfil (9) which implies that the inverse F-transform lies between interpretations of the generalized Mamdani FRB and the Łukasiewicz implicative FRB and therefore we talk about the additive interpretation of the FRB.

It is worth to mention that the introduced approach to an FRB interpretation using fuzzy partition avoids inconsistency, solves the problem of redundancy and does not require any complexity reduction algorithm (see [9]) because the complexity given by the number of basis functions is in an expert's hands since the very beginning and the algorithm builds just n rules.

If we want to generate the FRB automatically then we make some experiments and collect (measure) data. After the experiments, we are given a collection of crisp data (x_p, y_p) measured by some sensors and therefore we have to fuzzify the measured control actions y_p to get $\mathrm{Y}_p(y)$.

4 DYNAMIC ROBOT CONTROL

We deal with a dynamic robot control. The robot's task is to pass through any chosen corridor. The robot is equipped with a pivoted ultrasound sensor which provides imprecise information about his positions between walls.

For such purpose, we decided to use double-input-single-output FRB system where the input variables are: e - relative distance to a center of a corridor and Δe - change of the distance e in time. The output variable (control action) is the turning radius y of the robot.

As the title of this paper indicates, we solved the given problem with help of fuzzy approximation, namely the F-transform. During few experiments when the robot was driven manually by a human driver we were collecting data $(e_p, \Delta e_p, y_p)$ for $p = 1, \ldots, q$. The control actions y_p were fuzzified to get the fuzzy sets $\mathrm{Y}_p(y)$.

This data were proceeded the F-transform algorithm as described above to obtain fuzzy sets $\mathrm{F}_{ij}(y)$ - the components of the F-transform w.r.t. basis functions $\mathrm{A}_i(e), \mathrm{B}_j(\Delta e)$ (numbers and shapes chosen expertly). The whole algorithm led to the fuzzy relation $\mathrm{R}^F_{n,m}(e, \Delta e, y)$ which approximates the behaviour of the human driver.

The system can be viewed as the FRB system composed of the rules

$$\text{IF } e \text{ IS } \mathcal{A}_i \text{ AND } \Delta e \text{ IS } \mathcal{B}_j \text{ THEN } y \text{ IS } \mathcal{F}_{ij}, \tag{11}$$

with the additive interpretation given by (8). This FRB system together with the *center of gravity* deffuzification method was used for driving the robot through different corridors. From the robot's behaviour was clearly visible in which situations the system is not learned sufficiently.

Then, it is much easier for an expert to specify his knowledge for such cases than to build the whole FRB expertly from the very beginning. We added six elements to training data with linguistic fuzzy sets (see [2]) for the output variable i.e. $(e_{q+r}, \Delta e_{q+r}, \mathrm{Y}_{q+r}(y)),\ r = 1, \ldots, 6$. Then the whole algorithm of the F-transforms computed the components F_{ij} again and these components aggregated both types of information - experimental data and expert knowledge.

The new modified FRB system was found out to be appropriate and was many times sufficiently checked in different corridors. Based on the results of the practical testing, the number 7 of basis functions on both axes was found to be sufficient. It means that the robot was driven by 49 rules with the additive interpretation.

5 CONCLUSION

We recalled the basic definitions of the F-transform technique introduced in [3] and developed and applied to many problems in [4, 5, 7, 11, 12]. This technique is now applied to a continuous fuzzy relation and connected to the problematic of the fuzzy control and the automatic generation of fuzzy rules. Concrete application, dynamic robot control, was presented as an appropriate one for the fuzzy approximation approach, namely the F-transform.

The whole adaptation algorithm based on fuzzy approximation of a human driver's control and consecutive expert knowledge expressed in the human language performed well and required significantly short time since the very beginning till the very end of adaptation.

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