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Abstract

This paper deals with a dynamic robot driven by a fuzzy controller. The robot is supposed to be controlled in such a way to be able to go through an arbitrary corridor and to keep its motion as close to the middle of the corridor as possible. A special concept of Linguistic Fuzzy Logic Controller has been chosen and successfully implemented for a solution of this problem.
Keywords: Fuzzy control, Fuzzy modeling, Rule based-systems, Fuzzy sets, Fuzzy inference, Robot control.

1 Introduction

This paper describes the use of the software system LFLC (Linguistic Fuzzy Logic Controller) (Dvořák et al. (2003)) for the problem of the control of a dynamic robot. The task is the following: we have a corridor of an arbitrary shape. The robot is able to measure the distance to the walls of the corridor by means of an ultrasound sensor. Then, the robot should be able to go through the corridor and to keep itself in the middle of it, if possible.

We solve this problem by means of LFLC using the so-called perception-based logical deduction (see Novák and Perfilieva (2004)). This method relies on the conversion of the observed value/values to linguistic *perception* (Zadeh (2000)). This perception is in the form of so-called linguistic evaluating expression (Novák et al. (1999); Dvořák and Novák (2004)). Then, this perception is matched with the perceptions occurring in the antecedents of IF-THEN rules forming the rulebase of the fuzzy controller. Consequently, the rule which matches best the observation is selected and logical deduction is performed. Finally, the defuzzified value is found. The linguistic accent in LFLC 2000 guarantees that the robot acts in the way analogous to the human way of thinking. Actions the robot undertakes are performed in a similar way as actions of some human being with the knowledge expressed by the appropriate linguistic description in the form of IF-THEN rules.

The LFLC software system has already been used in industrial applications (Novák and Kovář (2000)), controlling e.g. a large aluminium furnace. This furnace has relatively slow reactions to control actions. Here we demonstrate the usefulness of LFLC in a different type of problem.

The paper is organized as follows: In section 2 we describe the problem in detail. Section 3 brings the material about perception-based logical deduction, the LFLC system and several related topics. In Section 4, the realization and results are presented. Finally, Section 5 contains conclusions and the outline of future work.

2 The problem and its description

Our task is to implement such automatic mechanism of cornering into the dynamic robot (see Figure 1) that the robot should be able to move through any possible corridor. At first stage, the problem of a simple corridor with no crossing and no obstacle has been solved.

The mechanism should be realized in a form of fuzzy controller, i.e. a controller working with so called fuzzy rule-based system consisting of a finite set of m IF-THEN rules in the following form

$$\text{IF } X_1 \text{ is } \mathcal{A}_1 \text{ AND } \ldots \text{ AND } X_n \text{ is } \mathcal{A}_n \quad \text{THEN } Y \text{ is } \mathcal{B}. \tag{1}$$

For more details, see Section 3.

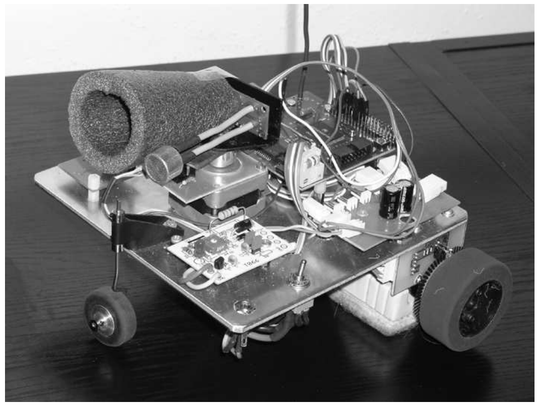

Figure 1: Dynamic robot

At first, let us describe the main features and properties of the robot. Simply written, the robot could be described as a square metal plane with three wheels: a front one which is free (movable) and two back ones. The back wheels are powered by their own electric engines. It allows the robot to turn the wheels with different speeds. There is an incremental sensor at each back wheel measuring a number of its revolutions. It gives us an information about a distance covered by the robot.

At the top of the robot, there is a rotating ultrasound sensor measuring a distance to the corridor wall in a direction of the sensor. The robot measures a distance d_j on one hand side and, while moving through the corridor, the sensor rotates to measure a distance d_{j+1} on the other hand side. This gives the robot an approximate information about its position between the left and the right wall.

An approximate relative distance to the middle of the corridor at moment j is denoted by e_j and computed as

$$e_j = (-1)^{j+1} \left(\frac{2d_j}{d_j + d_{j-1}} - 1 \right). \tag{2}$$

It is the first input variable into a fuzzy controller. The second input variable is a change of this relative distance Δe_j defined as

$$\Delta e_j = e_j - e_{j-1}. \tag{3}$$

The robot is controlled in such a way that its velocity is constant. The mentioned input variables are the only ones input variables and hence, we have a double-input-single-output system. The output variable could be either a turning radius u or its change in time Δu. The first choice leads to a PD fuzzy controller:

$$\text{IF } e \text{ is } \mathcal{A}_i^e \text{ AND } \Delta e \text{ is } \mathcal{A}_i^{\Delta e} \text{ THEN } u \text{ is } \mathcal{B}_i, \tag{4}$$

while the second one leads to a PI fuzzy controller:

$$\text{IF } e \text{ is } \mathcal{A}_i^e \text{ AND } \Delta e \text{ is } \mathcal{A}_i^{\Delta e} \text{ THEN } \Delta u \text{ is } \mathcal{B}_i \tag{5}$$

where $1 \leq i \leq m$ and $\mathcal{A}_i$, $\mathcal{B}_i$ are evaluating linguistic expressions (see Section 3). A control action in each IF-THEN rule is chosen to make the robot moving as close to the middle of the corridor as possible.

3 Used tool - LFLC

As a tool, used for the purpose introduced above, a concept of *Linguistic Fuzzy Logic Controller* (LFLC) with its software implementation LFLC 2000 has been chosen. The concept of LFLC has been introduced as a specific one which differs from the other already known implementations by two aspects: semantics of certain natural language expressions is implemented in such a way that computer behaves as if "understanding" them, and linguistically formulated conditional statements are used in genuine logical deduction. For detail information see Dvořák et al. (2003).

3.1 Linguistic descriptions and their elaboration

The theoretical background of LFLC and the control of our robot lays in formal fuzzy logic in broader sense (FLb), which is an extension of that in narrow sense (FLn) (for the detailed presentation of both logics see Novák et al. (1999)). The theory provides elaboration of that part of the semantics, which consists of the so called *evaluating* and *conditional linguistic expressions*. The former are expressions such as "small, roughly medium, very big", etc. The latter are the well known fuzzy IF-THEN rules. These are usually gathered into sets called *linguistic descriptions* which take the form

$$\mathcal{R}_1 := \mathsf{IF}\ X\ \mathsf{is}\ \mathcal{A}_1\ \mathsf{THEN}\ Y\ \mathsf{is}\ \mathcal{B}_1$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\mathcal{R}_m := \mathsf{IF}\ X\ \mathsf{is}\ \mathcal{A}_m\ \mathsf{THEN}\ Y\ \mathsf{is}\ \mathcal{B}_m$$

where $\mathcal{A}_j, \mathcal{B}_j$ are the mentioned evaluating linguistic expressions. They characterize property of some features of objects, for example size, volume, force, strength, etc. Since usually we are not interested in the concrete objects and their features, we replace them by some real numbers which are then represented by the variables X and Y. Thus, values of X and Y represent, e.g., values of temperature, pressure, price, etc. The linguistic expression of the form 'X is $\mathcal{A}$' is called the *evaluating linguistic predication*.

Fuzzy IF-THEN rules serve as a basis for *approximate reasoning*, which is a method for finding a conclusion on the basis of the imprecise initial information concentrated in the form of linguistic description and some new information. There are two fundamental approximate reasoning methods:

(a) *Perception-based fuzzy logical deduction*, i.e. finding a formal conclusion when fuzzy IF-THEN rules are treated as linguistically characterized logical implications.

(b) *Fuzzy approximation of a function*, i.e. finding a function which approximates some only imprecisely known function, whose course is estimated using the linguistic description.

The interpretation of the linguistic description significantly depends on the above chosen method.

The usual implementations of approximate reasoning focus on the method (b). Our concept of LFLC implements both methods but its main strength lays in the method (a).

3.2 Fuzzy approximation of a function

In this case, each evaluating predication 'X is $\mathcal{A}$' is assigned some formula $A(x)$ of predicate fuzzy logic. The whole linguistic description is then assigned one of two special formulas called the disjunctive and conjunctive normal form.

The *disjunctive normal form* is the formula

$$\mathrm{DNF}(x, y) := \bigvee_{j=1}^{m} (A_j(x) \wedge B_j(y)). \tag{6}$$

In this case, each rule is assigned a conjunction of formulas $A_j(x)$ and $B_j(y)$ and all of them are joined by disjunction. We speak also about *functional interpretation* of the linguistic description.

The alternative possibility is the *conjunctive normal form*

$$\mathrm{CNF}(x, y) := \bigwedge_{j=1}^{m} (A_j(x) \Rightarrow B_j(y)). \tag{7}$$

In this case, each rule is assigned an implication between the formulas $A_j(x)$ and $B_j(y)$ and all of them are joined by conjunction. We speak about *logical* interpretation of the linguistic description. Recall, however, that the main goal is still fuzzy approximation of a function.

Both formulas (6) and (7) correspond to certain fuzzy relations after the following assignment. Let us consider a couple of sets[1]

$$w = \langle U, V \rangle \tag{8}$$

[1] In fact, the problem is more complex since we must precisely specify the language, the structure and assignments of all symbols from the language. For the purpose of this paper, we simplify the explanation. The interested reader is referred to Novák et al. (1999); Perfilieva (2000).

which will be taken as a model. In the practice, we usually consider U, V to be some closed intervals of real numbers (as mentioned above, these represent, e.g. temperature, angles, prices, etc.). Furthermore, each formula $A_j(x)$ is assigned a fuzzy set $A_{w,j} \underset{\sim}{\subset} U$ and $B_j(y)$ is assigned a fuzzy set $B_{w,j} \underset{\sim}{\subset} V$, $j = 1, \ldots, m$. Then the disjunctive normal form (6) is assigned a fuzzy relation $R_{DNF,w} \underset{\sim}{\subset} U \times V$ given by the membership function

$$R_{DNF,w}(u, v) = \bigvee_{j=1}^{m} (A_{w,j}(u) \wedge B_{w,j}(v)), \tag{9}$$

and the conjunctive normal form (7) is assigned a fuzzy relation $R_{CNF,w} \underset{\sim}{\subset} U \times V$ given by the membership function

$$R_{CNF,w}(u, v) = \bigwedge_{j=1}^{m} (A_{w,j}(u) \rightarrow B_{w,j}(v)) \tag{10}$$

where $a \rightarrow b = \min(1, 1 - a + b)$, $a, b \in [0, 1]$ is the so called Łukasiewicz implication[2]. Note that (9) is the well known Mamdani-Assilian formula used in most applications of fuzzy control.

Now, let some value $u_0 \in U$ be given (i.e. this is some precise measurement of, say, temperature, on the basis of which we should find a proper control action). Then using (9) or (10) we derive a fuzzy set $B_{u_0} \underset{\sim}{\subset} V$ with the membership function

$$B_{u_0} = \left\{ R(u_0, v) / v \mid v \in V \right\}. \tag{11}$$

The result of this kind of elaboration of fuzzy IF-THEN rules is an approximating function $f^A : U \rightarrow V$ given by the formula

$$f^A(x) = \mathrm{DEF}(B_x), \qquad x \in U \tag{12}$$

where DEF is a defuzzification function. Recall that the defuzzification function is a function DEF : $(\mathcal{F}(U) - \{\emptyset\}) \rightarrow U$ where $\mathcal{F}(U)$ is the set of all fuzzy sets on U.

It has been proven that every continuous function on a compact set can be approximated using either conjunctive or disjunctive normal form with arbitrary precision, independently on the chosen defuzzification methods (cf. Novák et al. (1999)). However, a certain classification of defuzzification methods can be provided and the best possible one is Center of Gravity Method. More about approximation properties can be found also in Perfilieva (2000).

3.3 Perception-Based fuzzy logical deduction

The most specific feature of LFLC is the possibility to realize a *perception-based logical deduction* when the rules are interpreted as *linguistically characterized* logical implications.

3.3.1 Linguistic aspect

In the concept of LFLC, we deal with the mentioned *evaluating linguistic expressions* (possibly with signs) which have the general form

$$\langle \text{linguistic modifier} \rangle \langle \text{atomic term} \rangle \tag{13}$$

where ⟨atomic term⟩ is one of the words "small, medium, big", or "zero" (possibly also arbitrary symmetric fuzzy number) and ⟨linguistic modifier⟩ is an intensifying adverb such as "very", "roughly", etc.

The linguistic modifiers in (13) are of two basic kinds, namely those with narrowing and widening effect. *Narrowing* modifiers are, for example, "extremely, significantly, very" and *widening* ones are "more or less, roughly, quite roughly, very roughly". We will take these modifiers as canonical. Note that narrowing modifiers make the meaning of the whole expression more precise while widening ones do the opposite. Thus, "very small" is more precise than "small", which, on the other hand, is more precise than "roughly small".

[2] Alternatively, it can be any residuation operation based on some continuous t-norm.

The meaning of each linguistic expression $\mathcal{A}$ has two constituents: the *intension* $\mathrm{Int}(\mathcal{A})$ and *extension* $\mathrm{Ext}(\mathcal{A})$ in some *model* (this is often called the *possible world*).

Intension of the linguistic expression is a formal characterization of the property denoted by it on the level of formal syntax. It can be interpreted as a fuzzy set of special formulas[3]. However, it is a rather abstract concept, which in concrete situation (context) determines some fuzzy set of elements. Mathematically this means that a model w is given whose support is some set U (taken usually as a closed interval of real numbers). Then the extension of $\mathcal{A}$ is some fuzzy set of elements $\mathrm{Ext}_w(\mathcal{A}) \underset{\sim}{\subset} U$, which is determined by its intension $\mathrm{Int}(\mathcal{A})$. Note that for each concrete situation, different model should be considered. However, intension is still the same.

Note that the concepts of intension and extension formalizes the following intuitive situation: we can speak about *high temperature*, *high pressure*, *high tree*, etc. But high temperature may mean 100°C at home or 1000°C in metal melting process, and similarly in other cases. This cannot be satisfactorily formalized without the mentioned concepts.

In the terminology used in LFLC, we speak about *linguistic context* in which the given evaluating expression is used since in the practice, it requires setting the *minimal* and *maximal* possible values which can be attained by the used variables.

Let us stress that the extensions of the evaluating expressions are fuzzy sets of the form of the so called S- and Π-curves, as is depicted on Figure 2. More on the formal theory of evaluating linguistic expressions can be found in Novák (2001); Novák et al. (1999).

3.3.2 Perception-based logical deduction

Unlike fuzzy approximation, where we deal with fuzzy sets in a model (i.e. on the level of semantics), logical deduction must proceed on syntax. Instead of the detailed formal description, we will demonstrate the behavior of the logical deduction on an example.

Let us consider a linguistic description consisting of two rules:

$$\mathcal{R}_1 := \mathsf{IF}\ X\ \mathsf{is\ sm}\ \ \mathsf{AND}\ Y\ \mathsf{is\ sm}\ \ \mathsf{THEN}\ Z\ \mathsf{is\ bi}$$
$$\mathcal{R}_2 := \mathsf{IF}\ X\ \mathsf{is\ bi}\ \ \mathsf{AND}\ Y\ \mathsf{is\ bi}\ \ \mathsf{THEN}\ Z\ \mathsf{is\ sm}$$

where sm and bi are abbreviations of linguistic expressions small and big, respectively. These rules are assigned intensions $\mathrm{Int}(\mathcal{R}_1), \mathrm{Int}(\mathcal{R}_2)$, which can schematically be written as

$$\mathrm{Int}(\mathcal{R}_1) = (\mathbf{Sm}_x \wedge \mathbf{Sm}_y) \Rightarrow \mathbf{Bi}_z \tag{14}$$
$$\mathrm{Int}(\mathcal{R}_2) = (\mathbf{Bi}_x \wedge \mathbf{Bi}_y) \quad \Rightarrow \mathbf{Sm}_z. \tag{15}$$

Furthermore, let X, Y, Z be interpreted in a model which will consist of three sets $U = V = W = [0, 1]$. Then small values are some values around 0.3 (and smaller) and big ones some values around 0.7 (and bigger). Of course, given the input, e.g. $X = 0.3$ and $Y = 0.25$ then we expect the result $Z \approx 0.7$ due to the rule $\mathcal{R}_1$. Similarly, for $X = 0.75$ and $Y = 0.8$ we expect the result $Z \approx 0.25$ due to the rule $\mathcal{R}_2$.

The value 0.3 is represented in the formal system by a certain intension $\mathbf{Sm}'_x$ and similarly, the value 0.25 is represented by $\mathbf{Sm}'_y$.

Then the inference rule of modus ponens is applied on $\mathbf{Sm}'_x$, $\mathbf{Sm}'_y$ and the implication (14). The result is the intension $\mathbf{Bi}'_z$. The latter is to be interpreted as some fuzzy set $B' \underset{\sim}{\subset} W$.

To obtain one concrete value, the resulting fuzzy set B' should further be defuzzified. However, we deal with evaluating linguistic expressions, whose interpretation has always one of the three possible forms depicted on Figure 1. Therefore, standard defuzzification methods such as COG do not work properly. Instead, we have developed a special method, which we call Defuzzification of Evaluating Expressions (DEE). This method classifies first the type of the membership function and then decides the defuzzification, as is depicted on Figure 2. There are two versions of the DEE method, namely *simple* which first classifies the resulting fuzzy sets in types "small", "medium" and "big" and then defuzzifies it using Last of Maxima, Center of Gravity or First of Maxima methods, respectively. The second one

[3] They have the form $\mathbf{A} := \{a_t/A_x[t] \mid t \in M\}$ where $A(x)$ is a formula, M is a set of constants and a_t is an evaluation of the instance $A_x[t]$. For the details — see Novák et al. (1999); Dvořák and Novák (2004).

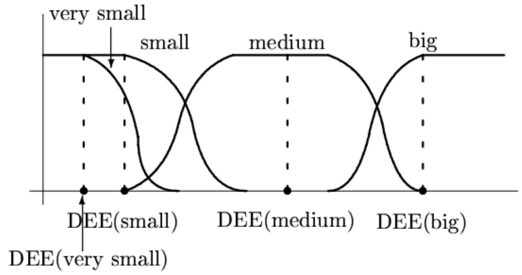

Figure 2: Form of fuzzy sets corresponding to the meaning of the evaluating linguistic expressions and the DEE defuzzification.

uses a sophisticated algorithm to choose a value close to these dependently on the specific shape of the membership function.

In our case, when the input is $X = 0.3$ and $Y = 0.25$ then both values correspond to "small" and thus, with respect to the rule $\mathcal{R}_1$, the result corresponds to "big" and thus, after its interpretation in the model and defuzzification using the DEE method, we obtain the result $Z \approx 0.7$, i.e. a value being intuitively big. In other words, we obtain the result which, on the basis of the form of the given rules, should be expected. Similarly, the input values $X = 0.75$ and $Y = 0.8$ would lead to the value $Z \approx 0.25$ due to the rule $\mathcal{R}_2$.

To summarize: in the case of fuzzy approximation, we form the special formulas DNF or CNF on the level of syntax, interpret them in some model and then find the approximation on the level of semantics only. In the case of perception-based fuzzy logical deduction we interpret the rules on the level of syntax, transform measurement also to this level, realize formal logical deduction and then interpret the result in some model.

3.4 Smooth logical deduction

Smooth logical deduction is a general modification of the inference/deffuzification process, which can be applied to a defuzzified result of an inference given by perception-based fuzzy logical deduction. The goal of the smooth deduction is to provide a *smooth* and *continuous* defuzzified inference result.

The main idea of the smooth logical deduction is based on so called F-transforms (see Perfilieva (2003)), which are in some sense applied as a filter to the original defuzzified inference result.

The original result is sampled over the whole input area by predefined step and a direct F-transform is applied to data obtained this way. The direct F-transform provides a vector representation of the original result. The final smooth and continuous deduction is obtained as an inverse F-transform. For more details we suggest Novák and Perfilieva (2004).

4 Realization

Due to the fact that fuzzy rules forming a fuzzy rule base for our problem are either in PD (4) or PI (5) shape we can start from some universal rule bases known for such problems. In most cases, we can start with such universal fuzzy rule base which is tuned with respect to system requirements afterwards. That is why universal fuzzy rule bases have been used in our attempts of an implementation of fuzzy logic to the discussed problem.

4.1 PD controller design

As we have mentioned in Section 2, there is the turning radius u used as the output variable in a PD controller. Perception-based fuzzy logical deduction implemented in LFLC 2000 has been chosen to be an appropriate tool for a realization of our approach. This allows us to use universal PD fuzzy rules like:

$$\text{IF } e \text{ is -medium AND}$$
$$\Delta e \text{ is zero THEN } u \text{ is +small.} \quad (16)$$

It follows that the linguistic description can be constructed symmetrically – we first design e.g. IF-THEN rules with positive linguistic expressions in the first antecedent variable. Then IF-THEN rules with negative expressions in this variable and expressions with inverted signs in remaining variables are added automatically. Finally, these rules are tuned manually, according to their behavior.

Such rules could be found either in respective literature or designed by an expert. After many experiments with the robot in different situations and different corridors, appropriate linguistic modifiers have been joined to the respective linguistic expressions. In that way, a linguistic expression for the turning radius u in presented rule (16) has been modified into +very small. Such tuning of the system has improved a behavior of the robot, i.e. helped to avoid all the crash moments as well as improved the smoothness of a motion.

Concerning the smoothness of the robots motion, a chosen defuzzification method should be discussed. While since the very beginning, COG defuzzification method has been founded to be inappropriate for our purpose, DEE method was a good candidate for our application. Although DEE performed well and could be suggested, an application of F-transforms (see Perfilieva (2003)) improved the smoothness enough to obtain practically optimal results. This approach is called *smooth logical deduction* and has been already mentioned in Section 3.4.

4.2 PI controller design

Although a PI controller was expected to be an appropriate one we have not managed to implement it successfully. The problem was hidden in technical deficits of the robot. More specifically, there was a too long period between the moments we measured the distances d_{j-1} and d_j. During that period the robot was still changing the turning radius what brought it into such a situation that it could not get out of there with inference based on rules where just a change of turning radius Δu is used as the output variable.

It does not mean that a PI controller is inappropriate in general. Similar robot with more sensors measuring at the same time would be more appropriate for testing control methods given by a PI controller. Moreover, the control action Δu could be changed to be a time-limited. It means that imaginary steering wheel would turn for a moment and would move back in time to be in the original - straight position before the next inference.

5 Conclusion

The problem discussed above has been finally solved by PD fuzzy controller with 54 IF-THEN rules, most of them symmetrically defined. The robot performed well and had no problems to go through bends about 180° when they were wide enough. Based on this behavior we can expect that the robot controlled by LFLC 2000 is able to execute more difficult tasks. For instance, one of further problems to be solved is avoiding obstacles as well as going through crossings. An execution of these additional tasks will require to add an input variables and, perhaps, one output variable - a speed of the robot. All of this is technically solvable by LFLC 2000 because the concept of LFLC 2000 offers wide choice of possibilities and improvements including a hierarchical structure of simpler fuzzy rule bases.

Another promising field of the future work lies in the use of learning algorithms which are able to construct a linguistic description from dataset obtained during the test ride of the robot through some training corridor. Appropriate learning algorithms are already implemented in LFLC 2000.

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