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Extended Abstract

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1 Introduction

In this presentation, we want to demonstrate some problems connected with defuzzification and chaining of rules in situations when there are more fuzzy inference engines connected in a hierarchical way. As the inference mechanisms in such a complex fuzzy systems we use so called fuzzy logic inference proposed and developed by V. Novák (see e.g. [12]). This inference method is theoretically well founded and proved itself to be useful also in practical applications.

By hierarchical fuzzy system we understand several fuzzy inference engines connected in such a way, that the output of one inference is input of another one (Figure 1).

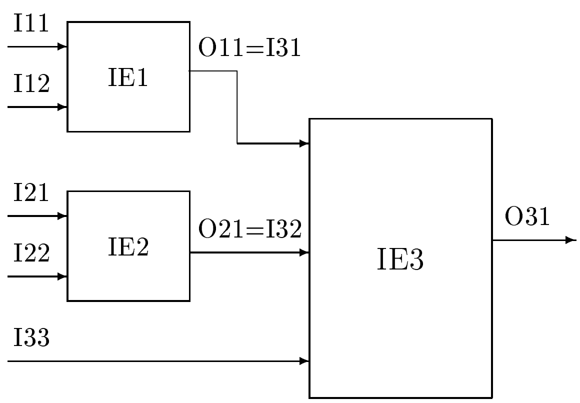

Figure 1: Example of hierarchical fuzzy system

Here, there are three inference engines IE1, IE 2 and IE3. IE1 and IE2 have two inputs, IE3 has three inputs, among them are outputs of IE1 and IE2. However, the structure of hierarchical fuzzy systems can be much more complicated. Such a systems can occur for example after a decomposition of some complicated fuzzy system with many antecedent variables. Then, it is almost impossible to describe that system by means of IF-THEN rules and it is necessary to divide the problem to some smaller blocks (inference engines with maximally three inputs). In these situations, there arise the following problem. As a result of one inference step, we obtain a fuzzy set. There is several possibilities we can choose as input to the second inference, namely: we can fetch this fuzzy set directly to input of the second inference engine and then we obtain inference with fuzzy inputs, which is computationally much more demanding. Beside that, it can also cause the outputs to be more and more "non-specific", i.e. output fuzzy sets tend to have the membership function close to the maximal membership degree on the whole universe of discourse after several inferences in such a hierarchical fuzzy system.

Therefore, it is necessary to search for other ways how to transmit output of one inference to input of the other. We can defuzzify output fuzzy set and use as a input to the second step defuzzified value. In

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this case we have to take into account the loss of information caused by defuzzification (instead of fuzzy set we use only one number). The third possibility is to perform linguistic approximation on output fuzzy set, obtain some linguistic expression which describes this set, and then construct new fuzzy set to it, which will be used as a input for the second inference engine. We will discuss all these possibilities in the following sections.

Let us now mention the relations of this investigation to the research field known as *chaining of rules*. There are several papers (e.g. [11, 14, 2, 6]) which study chaining of rules mainly from the following viewpoint: Let there be two linguistic descriptions with rules of the form IF X is A_i THEN Y is B_i, and IF Y is B_i THEN Z is C_i, $i = 1, \ldots, n$. A_i, B_i and C_i are some fuzzy sets. Then there are investigated conditions under which the linguistic description IF X is A_i THEN Z is C_i, $i = 1, \ldots, n$ gives the same results for all input fuzzy sets A' (papers [14, 2, 6]). These conditions turns out to put severe restrictions on possible use of t-norm and implication operators.

In paper [11] there are considered two linguistic descriptions IF X is A_i THEN Y is B_i, $i = 1, \ldots, n$ and IF Y is B'_j THEN Z is C_j, $j = 1, \ldots, m$ and authors constructed new linguistic description IF X is A'_i THEN Z is C_i, $i = 1, \ldots, n$, where fuzzy sets A'_i were computed from A_i, B_i and B'_i. It was proved there that outputs are the same for all inputs, provided that there is used Mamdami-Zadeh Max-Min or Max-t-norm inference. It is not known yet if a similar construction is possible for our fuzzy logic inference or other inference types. These results are very interesting and can be probably further extended, but it seems to be hard to use them in situations where more fuzzy inference engines are connected (Figure 1). Moreover, they are not working for fuzzy logic inference. Therefore, we try here to find some solutions to output-input transmission problem described above.

2 Inference with fuzzy inputs

The inference with fuzzy inputs was studied in previous papers [3, 5]. There were discussed problems with rapidly increasing computational complexity, which does not practically allow to use it for more than four antecedent variables. The fundamental formula which describes fuzzy logic inference mechanism is

$$B'y = \bigvee_{x_1 \in U_1, \ldots, x_n \in U_n} ((A'_1 x_1 \wedge \cdots \wedge A'_n x_n) \otimes \bigwedge_{j=1}^{r} ((A_{1j} x_1 \wedge \cdots \wedge A_{nj} x_n) \rightarrow B_j y)), \tag{1}$$

where n is number of antecedent variables, r is the number of rules in the linguistic description, $U_1, \ldots, U_n$ are universes of discourse of antecedent variables, $A_{1j}, \ldots, A_{nj}$, $A'_1, \ldots, A'_n$, B_j are fuzzy sets which represent the meanings of linguistic expressions in the antecedent part of the linguistic description, observations and linguistic terms in the consequent part of linguistic description, respectively. Construction of these meanings is described in the paper[13]. $\otimes$ and $\rightarrow$ are the Łukasiewicz conjunction and implication, respectively. The computational complexity of fuzzy logic inference with fuzzy inputs can be described by

$$\mathcal{C} = \mathcal{O}(rP^{n+1}), \tag{2}$$

where P is precision of discretisation (for the sake of simplicity, we assume that it is equal for all antecedent as well as succedent variables). Besides problems with computational complexity, there is also another problem: increasing fuzziness of outputs. When inputs don't match no rule fully, the result is fuzzy set which membership degrees are greater than zero for all $x \in X$. When such a set is used as a input for another inference step, the result will be now even more "fuzzy". Finally, it often occurs that as a result at the end of hierarchically chained inferences we obtain fuzzy set with all membership degrees close to one. Therefore, we can conclude that direct usage of output of one inference step as a input of the other is not advisible.

3 Defuzzification

Defuzzification is a procedure which assigned a crisp value to a given fuzzy set. It is necessary to use it in situations when numerical output of fuzzy system is required, e.g. in fuzzy control. However, it is clear that there must occur some information loss. There are a lot of defuzzification techniques described in literature. The most widely used is probably the COG (Center of Gravity) defuzzification, which can be described in case of continuous and integrable fuzzy sets defined on some universe of discourse X as a mapping $COG : [0,1]^X \to X$,

$$COG(A) = \frac{\int_X xAx\mathrm{d}x}{\int_X Ax\mathrm{d}x}. \tag{3}$$

However, when we use fuzzy logic inference, then this method has serious drawback which can be explained as follows. When there is no rule which fully fires for a given input, then result is a fuzzy set which membership degrees are greater than zero for all $x \in X$. When we perform COG on such a set, the result will be shifted to the universe center. Therefore, simple COG is not suitable for fuzzy logic inference. Center of Gravity method can be principally used here, but it needs to be modified in order to make it insensitive to those parts of membership function which bear no relevant information. Without going into detail here, this modification is based on computation of gravity center only from an α-cut of fuzzy set A, where α should be higher than the smallest membership degree of A. Formula (3) then becomes

$$ModCOG(A) = \frac{\int_{A_\alpha} xAx\mathrm{d}x}{\int_{A_\alpha} Ax\mathrm{d}x}. \tag{4}$$

Unfortunately, even this modified Center of Gravity method does not behave quite satisfactory. We tested it with "linear" linguistic description of the form If X is A_i THEN Y is B_i:

X	**Y**
extremely small	*extremely small*
very small	*very small*
small	*small*
.....	.....
medium	*medium*
.....	.....
big	*big*
very big	*very big*
extremely big	*extremely big*

This linguistic description has to model identity function, i.e. if we consider fuzzy inference engine with this linguistic description as a mapping $IE : [0,1] \to [0,1]$, then $IE(x) \cong x$ should hold for all $x \in [0,1]$. After some modifications of modified Center of Gravity Method, this requirement indeed holds. The usage of this defuzzification method in hierarchical fuzzy systems promises to bring satisfactory results.

4 Linguistic approximation

Linguistic approximation is, by definition, a procedure which construct appropriate linguistic description to a given fuzzy set [4]. The usage of it in hierarchical fuzzy systems is the following. As was mentioned in Section 2, one of drawback of using outputs of one inference step as a input of the second is increasing non-specificity caused by high membership degrees of output fuzzy sets on the whole universe. Linguistic approximation offers a solution: to the output of the first inference step we construct the linguistic expression (i.e. *very big*, *more or less medium or big* etc.). The we create a new "nice" fuzzy set to this expression and use it as the input of the second inference step. The linguistic approximation algorithm which is designed specially for fuzzy logic inference is described in [4]. The construction of meanings of obtained linguistic expressions, i.e. fuzzy sets which express vague notions as *very big*, *more or less small or medium* etc. is treated in detail in [13]. This solution of output-input transmission problem allows to get rid of problems with increasing "non-specificity" of outputs, there don't occur such information loss as in defuzzification method, but computational complexity can be high.

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