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## 1 Introduction

Image processing embodies a lot of problems with the great impact in the practical sphere. Nowadays, the main tools used in this field are connected to wavelet transform [5] (sometimes also windowed Fourier transform) and the related theory of frames, filters etc. One particular problem is connected to the focus measure (or alternatively blur measure). There already exists a number of focus measures of the given image based on the statistical characteristics, coefficients of wavelet transform, or comparison of low-level frequencies with the high-level ones, see [2]. The number of existing focus measures is huge and the choice of the most appropriate one is a rather difficult task.

The great popularity of wavelets in image processing follows from the fact that it can localize a given function in time (space) and frequencies. Partial results can be obtained by Perfilieva's Fuzzy Transform (FT) [3, 4] (Research report nr. 58) as well. And an iterative decomposition method introduced in [1] (Research report nr. 108[1]) under the title Full Fuzzy Transform (FFT) extends the potential of FT to various space and frequencies localizations. In this contribution, we present an alternative to the traditional focus measures for which we will consider coefficients of FFT. Moreover, we investigate their suitability in this particular problem area of image processing.

## 2 Basic concept of Full Fuzzy Transform

We will start with a brief overview of FFT. In [4], it has been shown that FT is the powerful tool for approximation of continuous functions, see Theorem 2. FFT stems from this fact and incorporates FT in approximations of residues of the given function on various levels.

**Fixed fuzzy transform**: In the sequel, assume function $f : X \mapsto Y,\ X, Y \subseteq \mathbb{R}$. Now we give the original definition of the F-transform taken from [3] for the 1-dimensional case.

Let $A_1, \ldots, A_k \subsetneqq X$ create Ruspini's fuzzy partition, i.e. $\sum_{i\in I} A_i(x) = 1$ for any $x \in X$.

**Definition 2.1** *Let $F_1, \ldots, F_k$ be given by $F_i = \frac{\int_a^b f(x)A_i(x)\,dx}{\int_a^b A_i(x)\,dx}$. The function*

$$T_{f,k}(x) = \sum_{i\in I} F_i A_i(x) \tag{1}$$

*will be called the fixed F-transform of $f$ w.r.t. $\{A_i\}_{i\in I}$.*

*Coefficients* $F_i$ of the F-transform serve us as a discrete representation of values of $f$ above supports of $A_i$'s. In fact, we are averaging all the values above intervals determined by $A_i$'s and its membership function is used as weights in this averaging.

**Full fuzzy transform**: At first, let us define the partial sums as follows:

$$S_n(x) = \sum_{i=0}^{n} f_{T,i}(x), \tag{2}$$

$$f_T(x) = f_{T,1}(x) + f_{T,2}(x) + f_{T,3}(x) + \ldots = \sum_{i=0}^{\infty} f_{T,i}(x), \tag{3}$$

[1]All research reports are available online at http:\\irafm.osu.cz

where

- $f_{T,0}(x)$ stands for arithmetic mean of $f(x)$ and error function $e_0 = f(x) - f_{T,0}(x)$.
- For $i \geq 1$,
$$f_{T,i}(x) = T_{e_{i-1},2^i},$$
represents FT of $e_{i-1}$ w.r.t. $\{A_i\}_{i \in J}$, $J = \{1, \ldots, 2^i\}$, where all integrals $\int_a^b A_i(x)\mathrm{dx}$ have the same values. Moreover
$$e_i(x) = e_{i-1}(x) - f_{T,i}(x).$$

**Remark 2.2** *Due to Theorem 2 [4], we find out that $S_n(x)$ converges uniformly to $f(x)$, whenever $f$ is continuous. Hence, $f_T(x) = f(x)$. In the case of discontinuity, we cannot cannot guarantee the previous equality.*

**Remark 2.3** *Note that we may increase the number of fuzzy sets in which we create fixed fuzzy transform arbitrarily and convergence is uniform. Also the starting approximation can be taken as fixed fuzzy transformation of a higher level (number of fuzzy sets $> 1$).*

**Remark 2.4** *Looking at the complexity of fuzzy transform, it is $\mathcal{O}(md2^d)$ (d – dimension, m – data size) that is the complexity of the same order as in the case of wavelet transform. Finally complexity of full fuzzy transform is $O(m^2 log_2 m)$.*

# 3 FFT-based focus measure

Below, we introduce new focus measure based on the coefficients of FFT related to the given functional representation of a particular image. Let us denote coefficients of $T_{e_{i-1},2^i}$ by $F_k^{(i)}$, where $k \in \{1, \ldots, 2^i\}$. Moreover, let $X \subset \mathbb{N}^2$ and $f : X \to \{1, 2, \ldots, 256\}$. Then, there exists $n$ such that $f_T(x) = S_n(x)$. We define *FTT-based focus measure* $\mathcal{M}(f)$ by the following formula:

$$\mathcal{M}(f) = \sum_{i=1}^{n} \sum_{k=1}^{2^i} |F_k^{(i)}|. \tag{4}$$

Recently, this focus measure has not been extensively investigated and its properties are not well known. Only the following observations can be obtained directly:

- For the image that consists of the one particular color, i.e., $f(x) = c, \forall x \in X$, we obtain $\mathcal{M}(f)$.
- Let $f_1, f_2$ represent one particular image of the different Gaussian blur. If $f_1$ is smoother (more blurred) than $f_2$ then $\mathcal{M}(f_1) \leq \mathcal{M}(f_2)$, because FTT coefficients of $f_1$ becomes smaller than those of $f_2$.

# 4 Future investigations

Since the coefficients of FFT represents local frequencies, they have good expressing power relating to the particular attributes of the given image that are recognizable on their basis. Other results in the field of focus measures lead us to a hypothesis that also (4) will serve as a good tool to measure (or at least relative) focus for some class of images. But we do not need to work with all coefficients, to use a part of them or choosing e.g. two levels of FT seems to be a promising way as well. Finally remark that this work is just at the beginning and it will be continually updated.

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