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# Full fuzzy transform and the problem of image fusion


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**Abstract**

The aim of this contribution is to present an alternative approach to the solution of image fusion problem and its comparison to the one based on the wavelet transform.

**Key words** Fuzzy transform, Image processing, Wavelet transform

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# 1 Introduction

Under the term image fusion problem we do understand an integration of complementary or multi-view information contained in the set of different images into one new image. The new image should be qualitatively better than each of the given images. The quality depends on the degradation or distortion relatively to the application requirements and this part of the problematic will not be the object of our study. Usually, wavelet transformation [3] is used to extract significant characteristics of the particular images.

The aim of this contribution is to present an alternative approach to the solution of image fusion problem. Instead of wavelet transform we consider fuzzy transform [1] and we investigate its suitability for the problem of image fusion. In [2], it has been shown that fuzzy transform is the powerful tool for approximation of continuous functions. In the sequel, we will work with one particular fuzzy transform that locally minimize weighted arithmetic variance. Therefore, it allows to extract different frequency w.r.t. local domains (similarly as wavelet transform).

# 2 Basic concept of full fuzzy transform

The main task is to find a formula using which we may express original function $f : X \mapsto Y$ using partial sums with an arbitrary precision, i.e.

$$f_T(x) = f_{T,1}(x) + f_{T,2}(x) + f_{T,3}(x) + \ldots = \sum_{i=0}^{\infty} f_{T,i}(x) = f(x). \tag{1}$$

Then we may transform functions $f_1, \ldots, f_p$ into $f_{1_T}, \ldots, f_{p_T}$ and operate on each level $i \in \mathbb{N}$ with $f_{1_{T,i}}, \ldots, f_{p_{T,i}}$. Moreover a composition needs to be specified which says us how to fuse functions.

Let us denote a partial sum

$$S_n(x) = \sum_{i=0}^{n} f_{T,i}(x). \tag{2}$$

**Fixed fuzzy transform**: Now we give the original definition of the F-transform taken from [1] for the 1-dimensional case. In the sequel, we assume that $X, Y \subseteq \mathbb{R}$.

Let $A_1, \ldots, A_k \subsetneq X$ create Ruspini's fuzzy partition [4], i.e. $\sum_{i \in I} A_i(x) = 1$ for any $x \in X$.

**Definition 2.1** *Let $F_1, \ldots, F_k$ be given by $F_i = \frac{\int_a^b f(x) A_i(x)\,dx}{\int_a^b A_i(x)\,dx}$. The function*

$$T_{f,k}(x) = \sum_{i \in I} F_i A_i(x) \tag{3}$$

*will be called the fixed F-transform of $f$ w.r.t. $\{A_i\}_{i \in I}$.*

*Coefficients* $F_i$ of the F-transform serve us as a discrete representation of values of $f$ above supports of $A_i$'s. In fact, we are averaging all the values above intervals determined by $A_i$'s and its membership function is used as weights in this averaging.

**Full fuzzy transform and fusion settings**:

- $f_{T,0}(x)$ stands for arithmetic mean of $f(x)$ and error function $e_0 = f(x) - f_{T,0}(x)$.
- For $i \geq 1$,
$$f_{T,i}(x) = T_{e_{i-1},2^i},$$
represents fuzzy transform of $e_{i-1}$ w.r.t. $\{A_i\}_{i\in J}$, $J = \{1, \ldots, 2^i\}$, where all integrals $\int_a^b A_i(x)\mathrm{dx}$ have the same values. Moreover
$$e_i(x) = e_{i-1}(x) - f_{T,i}(x).$$
- Fusion function $\kappa$ operates over the coefficients of fuzzy transforms of $f_{1_{T,i}}, \ldots, f_{p_{T,i}}$ at each level, e.g. it might be taken as
$$\kappa(x,y) = \begin{cases} y, & |x| \leq |y| \\ x, & \text{otherwise.} \end{cases}$$
- Fused function is given by $F_T(x) = \bar{f}_{T,1}(x) + \bar{f}_{T,2}(x) + \bar{f}_{T,3}(x) + \ldots = \sum_{i=0}^{\infty} \bar{f}_{T,i}(x)$, where $\bar{f}_{T,i}(x) = \sum_{i\in I} F_i A_i(x)$ for each $i$ and $F_1, \ldots, F_{2^i}$ are determined on the basis of coefficients of fixed fuzzy transformations $f_{1_{T,i}}, \ldots, f_{p_{T,i}}$ using $\kappa$.

**Remark 2.2** *Note that we may increase the number of fuzzy sets in which we create fixed fuzzy transform arbitrarily and convergence is uniform. Also the starting approximation can be taken as fixed fuzzy transformation of a higher level (number of fuzzy sets $> 1$). Moreover, there exist a lot of different possibilities how to specify $\kappa$, see [5].*

**Remark 2.3** *Looking at the complexity of fuzzy transform, it is $\mathcal{O}(md2^d)$ (d – dimension, m – data size) that is the complexity of the same order as in the case of wavelet transform. Finally complexity of the image fusion with using the fuzzy transform is $O(pm^2 log_2 m)$ (p – number of input images).*

**Example 2.4** *Let us assume two discrete functions $f_1, f_2 : I \mapsto [0,1]$ representing 191'th rows taken from different blurred images of Lenna.BMP.*

*Figure 1 illustrates the fixed fuzzy transformations (black lines) of $f_1$ and $f_2$ (grey lines) over the 128 fuzzy sets and Figure 1 shows the fusion on the basis of $\kappa$ (black line) compared with the ideal image row (grey line).*

**Example 2.5** *Analogously as in the case of one input variable functions, we may apply the technique of full fuzzy transform to the images, i.e. functions of two variables, see Figure 3.*

# 3 Conclusions

The experimental examples show that the full fuzzy transform might be used in the problematic of image fusion. In this contribution the particular fuzzy transform has been used. But important constituents of fuzzy transforms are those based on the operations creating residual lattice. These transforms can be employed in the analogous way. Of course the coefficient of these transforms carry different information about given function. Hence, the qualitative criterion (function operating over the coefficients) needs to be adjusted.

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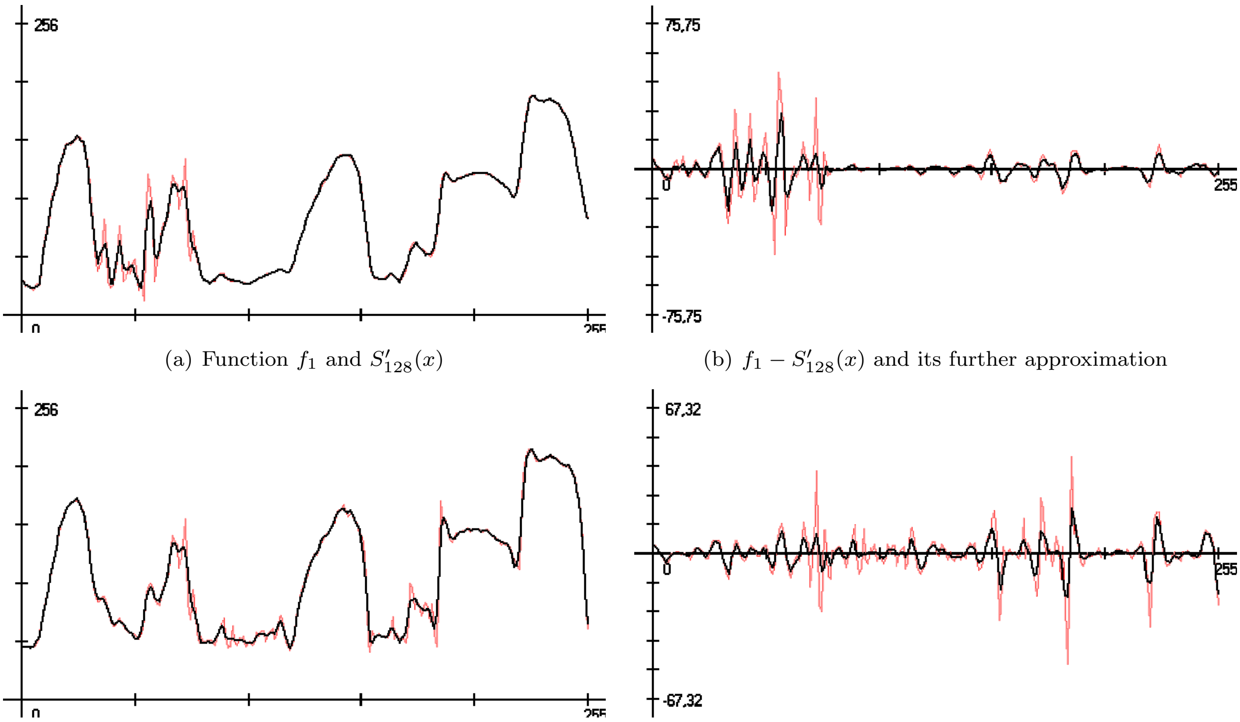



(a) Function $f_1$ and $S'_{128}(x)$

(b) $f_1 - S'_{128}(x)$ and its further approximation

(c) Function $f_2$ and $S''_{128}(x)$

(d) $f_2 - S''_{128}(x)$ and its further approximation

Figure 1: Functions to be fused (grey) and their approximations (black).

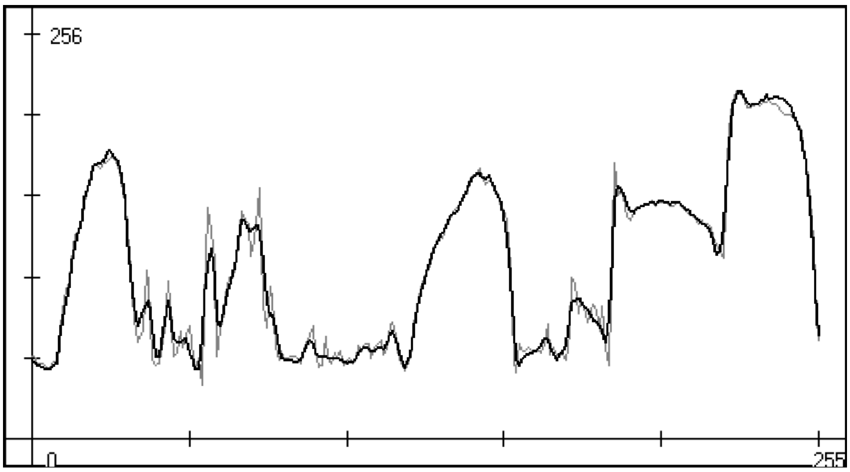
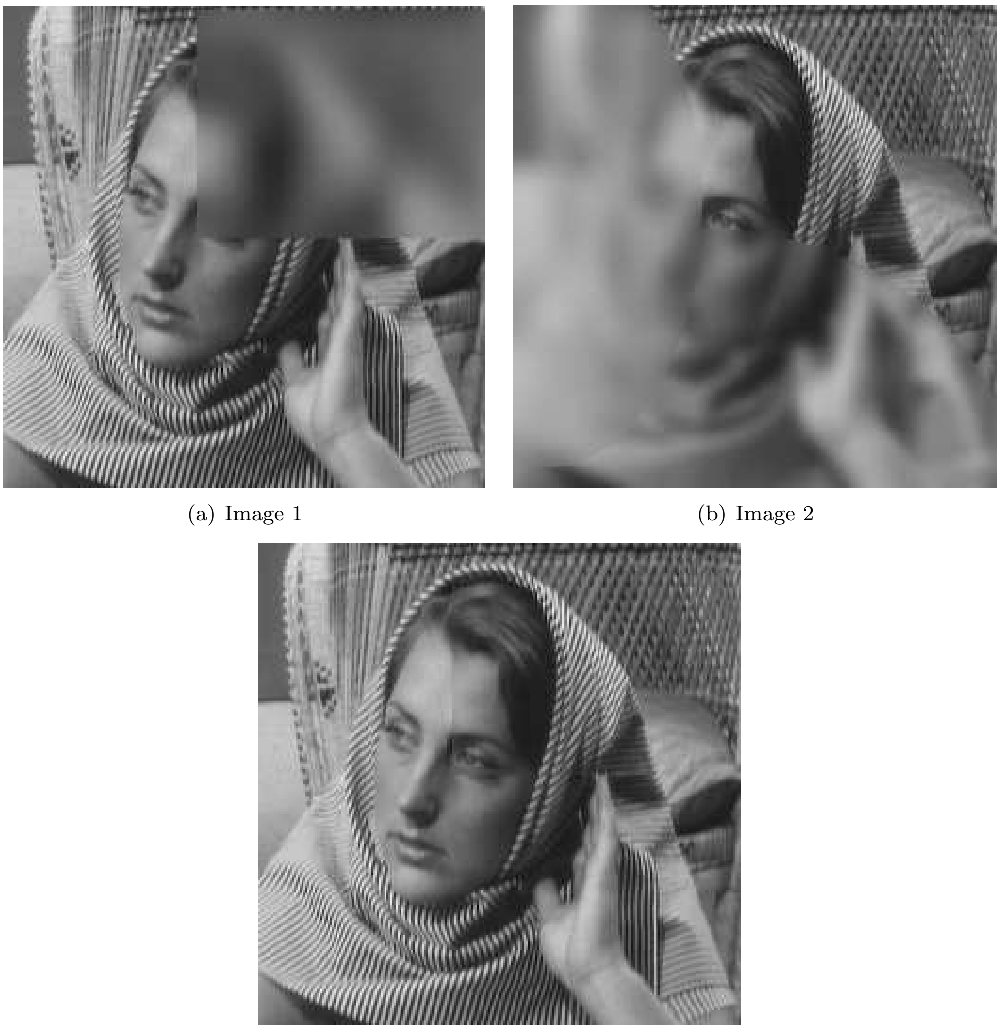



Figure 2: Fused function (black) after 5 iterations and an ideal function (grey).



(a) Image 1

(b) Image 2

(c) Fusion of image 1 and 2.

Figure 3: Example of image fusion.

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