University of Ostrava

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Dagmar Plšková

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University of Ostrava
Institute for Research and Applications of Fuzzy Modeling
30. dubna 22, 701 03 Ostrava 1, Czech Republic

tel.: +420-59-6160234 fax: +420-59-6120 478
e-mail: dagmar.plskova@osu.cz

Fuzzy Transform in Geological Applications

Dagmar Plšková

Institute for Research and Application of Fuzzy Modeling,
30. dubna 22, 701 03 Ostrava 1, Czech Republic,
dagmar.plskova@osu.cz

In this paper we deal with applications of the fuzzy (F–) transform method in geology. This method belongs into the area of fuzzy approximation methods. The F–transform method is used to solving complicated differential equations that describe coral reef growth.

K e y w o r d s: F–transform, basic functions, geological applications.

2000 Mathematics Subject Classification: 68T37

1 Introduction

The F–transform is a known technique for an approximation of continuous functions. The method consists of the direct F– transform and the inverse F–transform. This method has been developed by I. Perfilieva (see in [3]) and has been used for solving ordinary differential equations (see in [4]).

One of possible applications of the fuzzy transform is modeling of geological reliefs. Concretely, we will deal with one dimensional and two dimensional models of a coral reef growth due to knowledge of a sea level history. And vice versa, we will model sea level history due to knowledge of coral reef increments in time. The models are based on geological hypotheses. We use knowledge from [1] and [2]. This paper extends and specifies [4]. In addition a technical background of this applications is published there. The paper focuses on the other problem coral reef too.

2 Fuzzy Transform method

In this section we introduce the F–transform method and its using for solving ordinary differential equations.

2.1 Basic definitions

We take a closed bounded and real valued interval $[a, b]$ as a universe. First of all we define a uniform fuzzy partition of $[a, b]$.

Definition 1
Let $a = x_1 < x_2 < \ldots < x_n = b$ be an equidistant partition of $[a, b]$, that is $x_k = a + (k-1)\Delta$, $k = 1, \ldots, n$, where $\Delta = \frac{b-a}{n-1}$ is the discretization parameter of the partition. We say that basis functions (fuzzy sets) *$A_k : [a, b] \to [0, 1]$, $k = 1, \ldots, n$ form a* uniform fuzzy partition *of $[a, b]$ if they satisfy the following conditions:*

- $A_k(x_k) = 1$, $A_k(x) = 0$ *if* $x \notin (x_{k-1}, x_{k+1})$, $k = 1, \ldots, n$, *where* $x_0 = a$ *and* $x_{n+1} = b$
- A_k, $k = 1, \ldots, n$ *is continuous on* $[a, b]$
- A_k, $k = 2, \ldots, n$ *increases on* $[x_{k-1}, x_k]$ *and* A_k, $k = 1, \ldots, n-1$ *decreases on* $[x_k, x_{k+1}]$
- $\sum_{k=1}^{n} A_k(x) = 1$ *for all* $x \in [a, b]$.

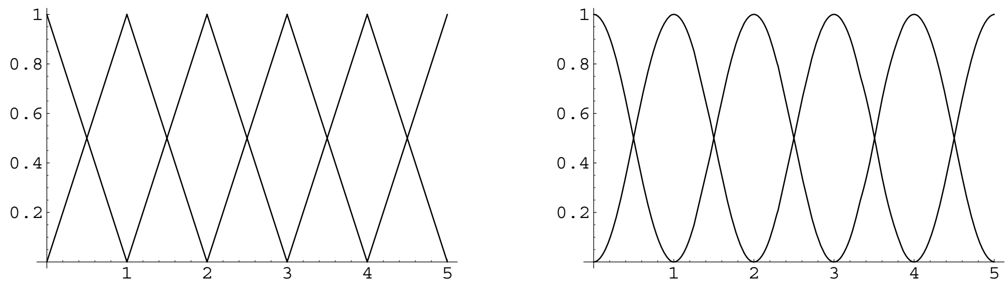

Figure 1: Examples of fuzzy partitions with triangular and sinusoidal basic functions.

In Figure 1, there are examples of uniform fuzzy partitions.

The F-transform method is created by the direct and the inverse F-transform.

Definition 2
Let $A_1, \ldots, A_n$ be basic functions which form a fuzzy partition of $[a, b]$ and let f be a function from the Lebesque's space $L_1(a, b)$. We say that the n-tuple of real numbers $[F_1, F_2, \ldots, F_n]$ given by

$$F_k = \frac{\int_a^b f(x)A_k(x)dx}{\int_a^b A_k(x)dx}, \qquad k = 1, \ldots, n.$$

is the direct F-transform of the function f with respect to $A_1, \ldots, A_n$.

The values F_k, $k = 1, \ldots, n$ of the direct F–transform approximate the values $f(x_k)$.

Definition 3
Let $[F_1, F_2, \ldots, F_n]$ be the direct F-transform with respect to the fuzzy partition $A_1, A_2, \ldots, A_n$. The function

$$f_{F,n}(x) = \sum_{k=1}^{n} F_k A_k(x)$$

is called the inverse F-transform.

By the inverse F-transform we obtain the approximation of the original function.

2.2 The generalized Euler method

In this section we show how the F-transform method can be used for solving differential equations. Suppose that the Cauchy problem

$$\begin{aligned} y'(x) &= f(x, y) \\ y(x_1) &= y_1 \end{aligned} \tag{1}$$

is given.

Let us choose the uniform fuzzy partition of the interval $[x_1, x_n]$ with respect to basic functions $A_1, \ldots A_n$ and with the discretization parameter $\Delta = \frac{x_n - x_1}{n-1}$.

Theorem 1
Let the Cauchy problem with twice differentiable parameters be transformed by applying the F-transform with respect to the basic functions $A_1, \ldots A_n$ to both sides of a given differential equation (1). Then the components of the F-transform of y with respect to the same basic functions can be found approximately from the following system of equations

$$\begin{aligned} Y_1 &= y_1 \\ Y_{k+1} &= Y_k + h\widehat{F_k}, \; k = 1, \ldots, n-1, \end{aligned}$$

where

$$\widehat{F_k} = \frac{\int_a^b f(x, Y_k) A_k(x) dx}{\int_a^b A_k(x) dx}.$$

The order of the local approximation error is $O(\Delta^2)$.

The proof has been performed by I. Perfilieva (see [4]).

3 F-transform in geological applications

In this section we show how the F-transform method can be used in geology applications. Concretely we will simulate coral reef growth. Some of the results of this work were published in [4].

First of all we demonstrate a problem of coral reef growth in one dimensional model. We suppose that the sea level history is given. The dependence of coral reef growth on time is describe there. Then we deal with two dimensional model. The dependence of coral reef growth on time and distance from a bank. Finally we solve an inverse problem. In this case coral reef growth increments are given and the sea level history is looked for.

3.1 One dimensional model

In this case we will simulate coral reef growth of the Alcran reef near Mexico. It is known that growth of coral reef depends on salinity, temperature, intensity of light, etc. We will only consider the dependence on intensity of light. It is clear that light intensity depends on depth: the greater depth the smaller light intensity and therefore coral reef growth is smaller too. On the other hand the coral reef does not rise above sea level.

The model has been described by the following equation from geologists Bosser and Schlager:

$$\frac{dh(t)}{dt} = G_m \tanh\left(\frac{I_0}{I_k} exp\left(-k[(h_0 + h(t)) - (s_0 + s(t))]\right)\right), \tag{2}$$

where $h(t)$ is the growth increment, h_0 the initial height, G_m the maximal growth rate, k the extinction coefficient, I_0 the surface light intensity, s_0 the initial sea level position, I_k the saturating light intensity and $s(t)$ the sea level variation.

We gain the enter parameters into the equation from geological measurements. The entries for our area are following: $G_m = 12\, mm\, yr^{-1}$, $I_0 = 2000\, \mu\, E\, m^{-2}\, s^{-1}$, $I_k = 400\, \mu\, E\, m^{-2}\, s^{-1}$, $k = 0,15\, m^{-1}$, $h_0 = 0\, m$, $s_0 = 0\, m$.

We suppose that studied area was overflowed about 10000 years ago. Further we consider that the coral reef growth started 7800, 7900, 8000 and 8100 years ago. The initial conditions for the considered cases are the following:

$$\begin{array}{ll} h(8100) = 33,6m & h(8000) = 33,6m \\ h(7900) = 33,6m & h(7800) = 33,6m \end{array}$$

Assumed sea level history is depicted in Figure 2.

The differential equation (2) was solved by the generalized Euler method. For the F-transform method we use the triangular basis functions (see Figure 1) and with the discretization parameter $\Delta = 100\, years$. The approximated coral reef growth $h(t)$ for the mentioned initial conditions are depicted in Figure 2. Note that the coral reef growth is broken on the sea level.

For numerical realization was used software Mathematica.

3.2 Two dimensional model

We simulate the growth of the complete coral reef. A geological hypothesis about the original slope of the reef is used there. Due to this fact we obtain the dependence between the depth of the sea level and the distance from a bank. For changing initial depth we will consider one dimensional model of the

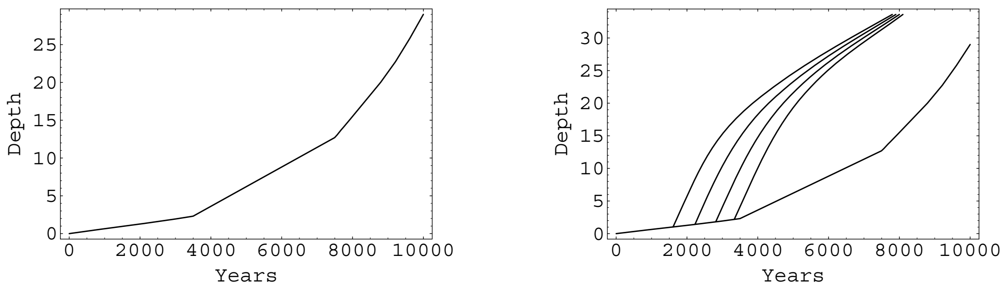

Figure 2: On the left side - sea level history and on the right side - coral reef growth of Alcran reef

coral reef growth given by the equation (2). Thus the two dimensional model of the coral reef growth is approximated there by multiple using the one dimensional one.

We focus on the reef near the island Belize in Mediterranean Sea. We have geological data from geological studies again. The date are following: $G_m = 0,005\, mm\, yr^{-1}$, $I_0 = 2000\, \mu\, E\, m^{-2}\, s^{-1}$, $I_k = 250\, \mu\, E\, m^{-2}\, s^{-1}$, $k = 0,05\, m^{-1}$, $h_0 = 0\, m$, $s_0 = 0\, m$. We approximately describe the sea level by the function $s(t)$ again. This function is obtained from numerical values from [1] by the inverse F–transform method. The sea level is illustrated in Figure 3.

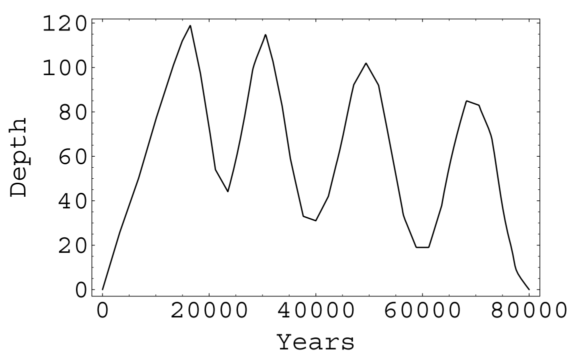

Figure 3: Sea level history

It is possible to see in Figure 3 that the sea level often oscillated. Therefore the coral reef rose above the sea level several times and growth was stopped until a repeated overflow. So the generalized Euler method for solving of the differential equation (2) must be several times restarted.

This model suppose that the coral reef growth started before 80000 years. The initial depth $h(80000)$ is taken after 10 meters from 0 to 200 meters. For example the coral reef growth for initial depth $h(80000) = 150\, m$ meters is depicted in Figure 4. For this initial depth we must solve the equation (2) thrice. The restarted initial conditions were $h(25000) = 60\, m$ and $h(7000) = 47,5\, m$. Horizontal parts of the curve mean that the reef was above the sea level and it did not rise.

By described way we obtain 21 curves for 21 initial depths. Further we use geological hypothesis that the original slope of the reef was $50°$. Therefore we can express dependence of depth of the sea on a distance from a bank. So we can model shape of the coral reef between the present time and 80000 years ago because we know the increments of reef in the determined depth in arbitrary time $t \in [0, 80000]$. For example in Figure 4 we have $h(40000) = 80\, m$. It means that the reef increment for initial depth $150\, m$ was $70\, m$ for 40000 years.

Gradual evolution of reef near the island Belize is illustrated in Figure 5. Each curve in Figure 5 represents a coral reef increment for 1000 years. First curve (on the left) is linear and represents the original slope of the reef. The last one (on the right) models the present shape of the reef.

This result is similar with the result of Bosser and Schlager [1], which solved this problem by Runge-

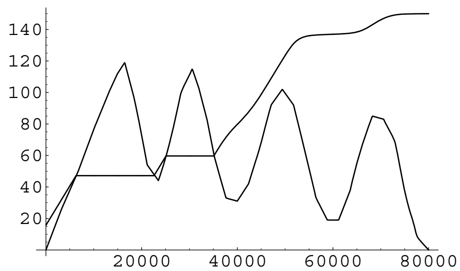

Figure 4: One dimensional model with the initial condition $h(80000) = 150$ meters.(horizontal axis - years and vertical axis - depth of the sea)

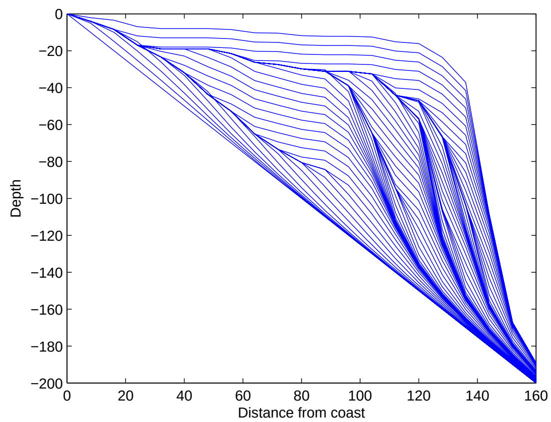

Figure 5: Coral reef growth

Kutta method of the fourth order and the one of Demicco and Klir [2], which solved described problem by fuzzy linguistic model.

For numerical realization were used softwares Mathematica and Matlab.

3.3 Inverse problem

Now we use stratigraphic measured section where we are given the growth increment $h(t)$ and find sea level history $s(t)$.

The input data (Table 1) have been obtained empirically from a vertical measured section of an Upper Cambrian limestone from western Maryland. The section comprises a sequence of thickness of eight types of rocks. Each type corresponds to a certain water depth and therefore determines ancient sea level. Because sea level rises up and then goes down repeatedly, the total sequence of thicknesses of types of rocks can be divided into cycles characterizing one period of sea level rise. We use this fact for a mathematical model of $h(t)$. In Table 1 there are 226 rows. For the sake of simplicity we will assume that each row corresponds with the same time period $\triangle t$.

From the differential equation (2) we derive the relation for unknown sea level history s(t):

$$s(t) = \frac{1}{k} \ln\Big[\frac{I_k}{I_0}\, arctanh\Big(\frac{\triangle h}{\triangle t\, G_m}\Big)\Big] + h(t),$$

where $\triangle t = 1$. The other parameters in this equation are the same as in two dimensional model from the previous subsection. However the graph of $s(t)$ is too oscillating. Therefore we use the F−transform method with sinusiodal-shaped basis function to smoothing this function. Due to the smoothing process

Rock type	Rock thickness	Cycle number	Cycle thickness
4	1.2	1	1.2
1	0.4	1	1.6
3	0.1	1	1.7
4	5.5	1	7.2
3	0.1	2	0.1
⋮	⋮	⋮	⋮
4	2.0	45	6.4
3	2.7	45	9.1

Table 1: Example of input data

we can better determine the period when the sea level rises up and then goes down. The result is shown in Figure 6.

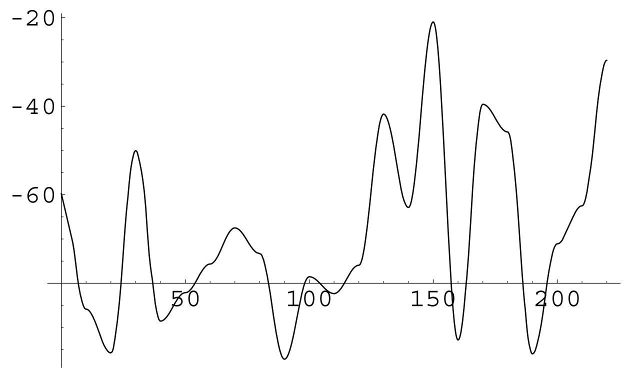

Figure 6: Sea level

For numerical realization was used software Mathematica.

4 Conclusion

This paper describes the F–transform method and shows how the method was used successfully in geological applications.

Computation simplicity, stability with respect to changes in initial data and good accuracy, comparable with analogous numerical methods are the advantages of the F–transform method. For instance the F–transform compared with finite difference method is characterized by robustness e.g. stability in relation to data faults [5].

In further investigation we would like to apply the F–transform method to other applications in geology.

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Dagmar Plšková (Mgr) is research assistant at the Institute for Research and Applications of Fuzzy Modeling and PhD student in applied mathematics at the University of Ostrava. Her supervisor is Prof. Irina Perfilieva.