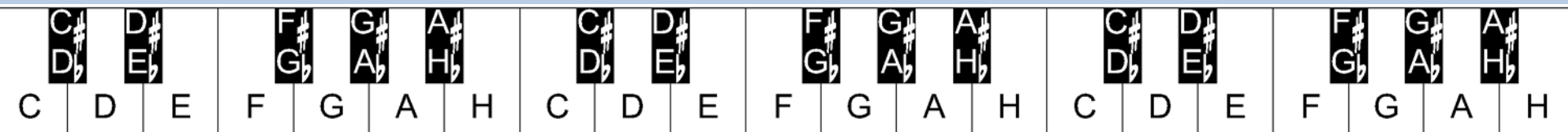


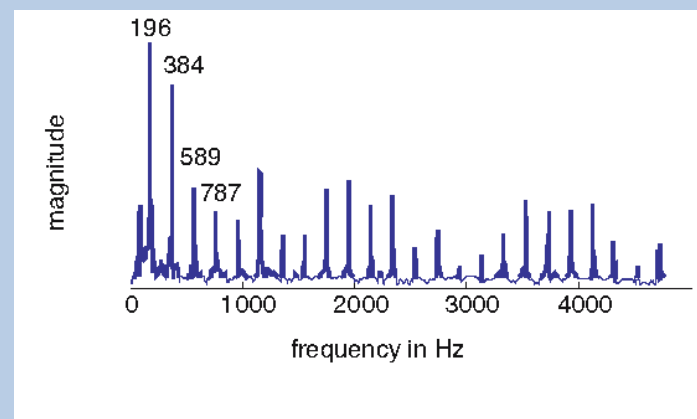
# Analysis of dissonance curves

Ladislav Šipeky



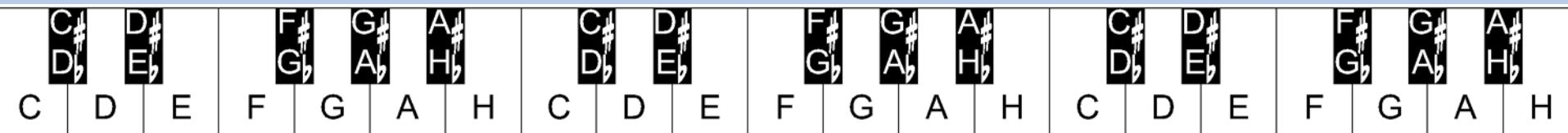
## Real tones

- Hermann von Helmholtz (1821 - 1894)
- Result of Fourier analysis



- Sum of individual sine waves

$$\sum_{n=1}^{\infty} f_n \sin(F_n t) \quad - \text{simplified}$$

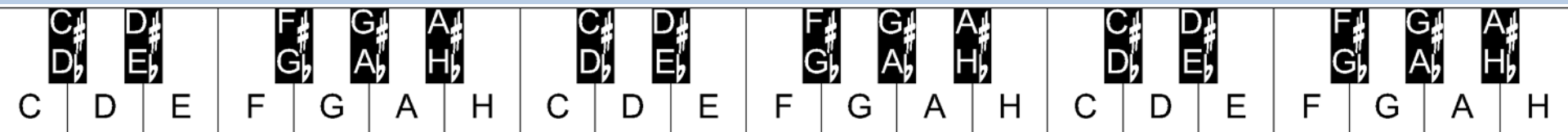


## Experiment of Reinier Plomp and Willem Levelt

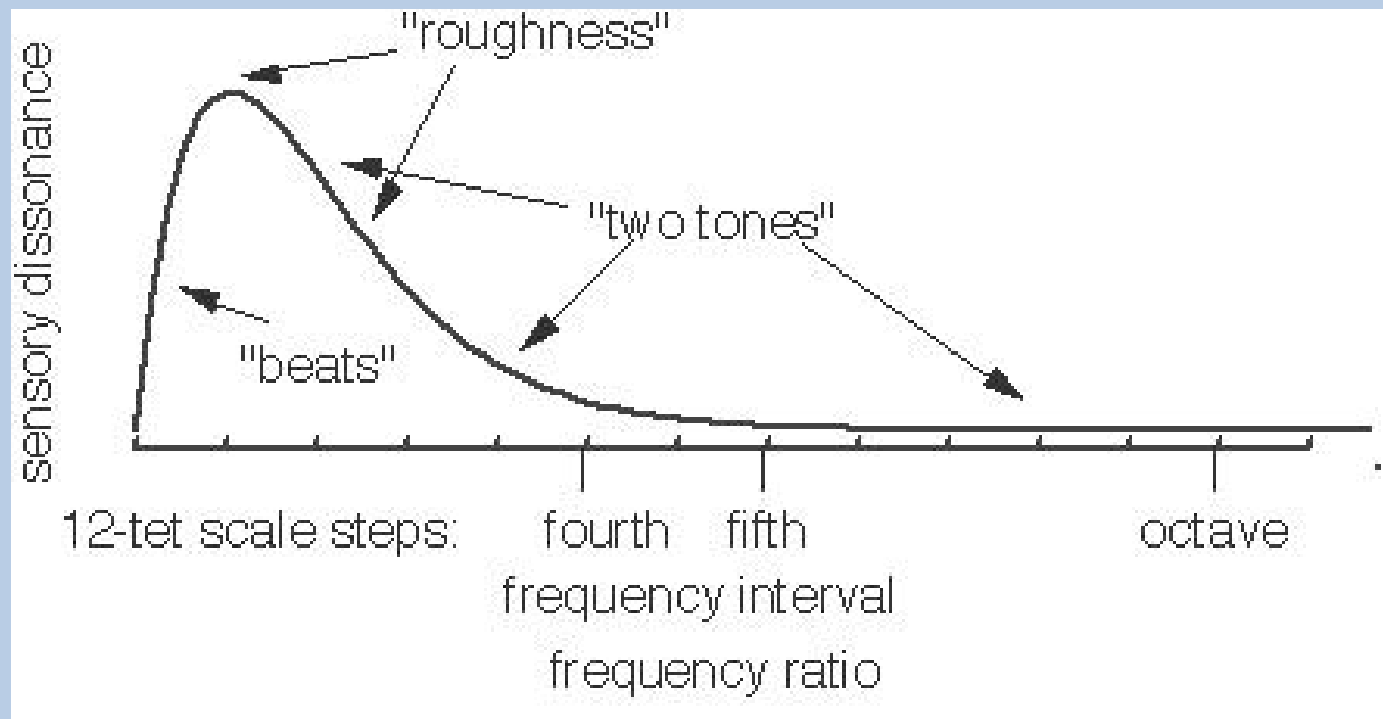
- Individual sine wave pairs - Pure tones
- Subjects without musical education
- Evaluation of dissonance (1-7 scale)

Plomp, R.-Levelt, W.J.M.:

Tonal Consonance and Critical Bandwidth, J. Acoust. Soc. Am. Vol. 38, Issue 4, pp.548-560, 1965



## Dissonance curve



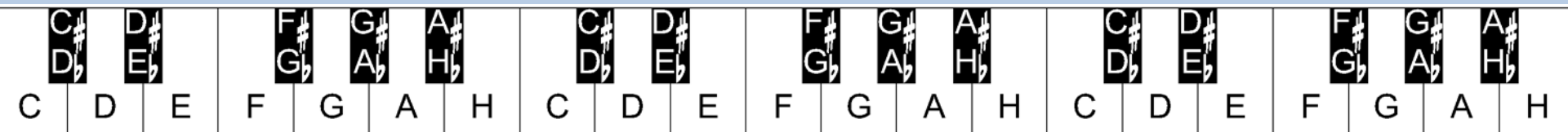
Taken from: <http://eceserv0.ece.wisc.edu/~sethares/consemi.html>

C <sub>♯</sub>	D <sub>♯</sub>	F <sub>♯</sub>	G <sub>♯</sub>	A <sub>♯</sub>	C <sub>♯</sub>	D <sub>♯</sub>	F <sub>♯</sub>	G <sub>♯</sub>	A <sub>♯</sub>	C <sub>♯</sub>	D <sub>♯</sub>	F <sub>♯</sub>	G <sub>♯</sub>	A <sub>♯</sub>
D <sub>b</sub>	E <sub>b</sub>	G <sub>b</sub>	A <sub>b</sub>	H <sub>b</sub>	D <sub>b</sub>	E <sub>b</sub>	G <sub>b</sub>	A <sub>b</sub>	H <sub>b</sub>	D <sub>b</sub>	E <sub>b</sub>	G <sub>b</sub>	A <sub>b</sub>	H <sub>b</sub>
C	D	F	G	A	C	D	F	G	A	C	D	F	G	A
				H					H					H

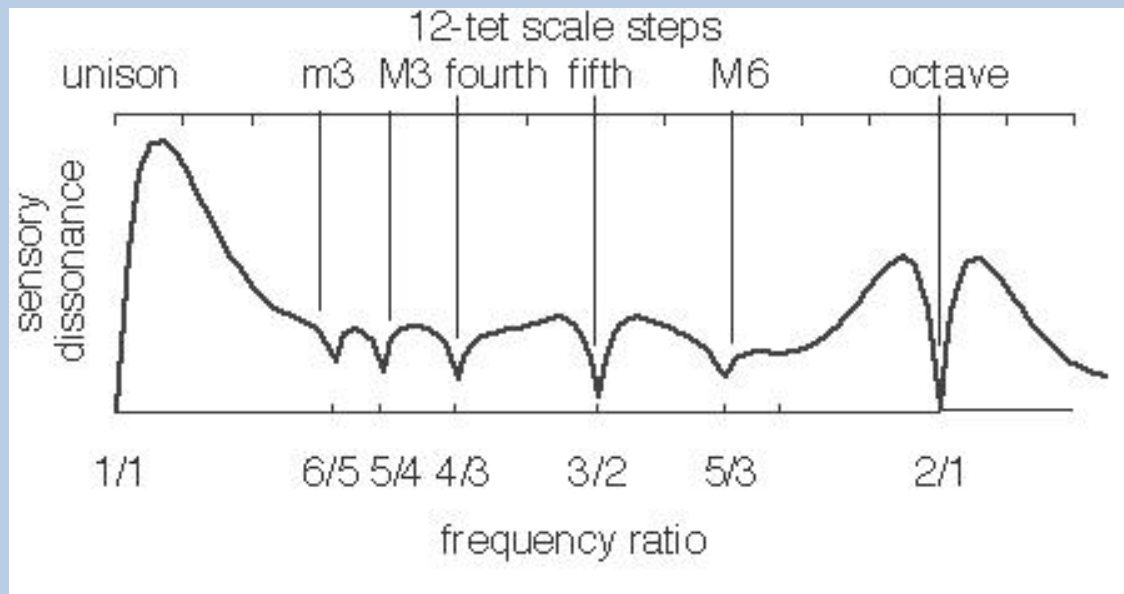
# Beats

Acoustical effect - interference of close frequencies

$$F_{beats} = |F_1 - F_2|$$



## Dissonance curve of „real“ tone



Taken from: <http://eceserv0.ece.wisc.edu/~sethahares/consemi.html>

C <sup>#</sup> D <sub>b</sub>	D <sup>#</sup> E <sub>b</sub>	F <sup>#</sup> G <sub>b</sub>	G <sup>#</sup> A <sub>b</sub>	A <sup>#</sup> H <sub>b</sub>	C <sup>#</sup> D <sub>b</sub>	D <sup>#</sup> E <sub>b</sub>	F <sup>#</sup> G <sub>b</sub>	G <sup>#</sup> A <sub>b</sub>	A <sup>#</sup> H <sub>b</sub>	C <sup>#</sup> D <sub>b</sub>	D <sup>#</sup> E <sub>b</sub>	F <sup>#</sup> G <sub>b</sub>	G <sup>#</sup> A <sub>b</sub>	A <sup>#</sup> H <sub>b</sub>						
C	D	E	F	G	A	H	C	D	E	F	G	A	H	C	D	E	F	G	A	H

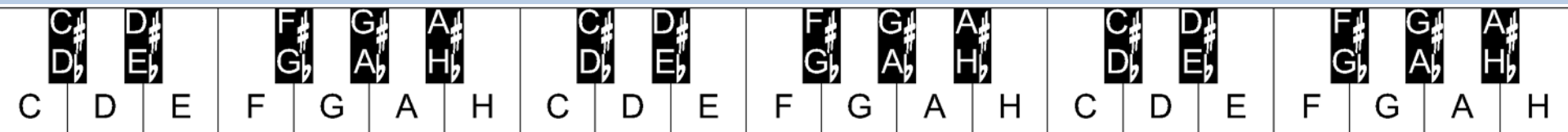
# Analysis of dissonance curves

- Synthetized sounds:

$$\mathbf{F} = (F_i, f_i) \quad \mathbf{G} = (G_j, g_j)$$

- Function:  $d(F, G, f, g)$

- Dissonance:  $D_f = \sum_{i=1}^n \sum_{j=1}^m d(F_i, G_j, f_i, g_j)$



## Various formulas

- Plomp-Levelt (1965) / Sethares (1999)

$$d(F, G, f, g) = \min(f, g) \left[ e^{-\alpha \frac{b(|F-G|)}{s_1 F + s_2}} - e^{-\beta \frac{b(|F-G|)}{s_1 F + s_2}} \right]$$

$$\begin{aligned} \alpha &= 3,5 \\ \beta &= 5,75 \\ s_1 &= 0,021 \\ s_2 &= 19 \end{aligned}$$

- Lafrenier (2001)

$$d(F, G, f, g) = \min(f, g) \left[ e^{-\left( \log \left( \frac{b(|F-G|)}{F} \right) \right)^2} \right] \quad d(F, G, f, g) = \min(f, g) \left[ |F - G| e^{-b(|F-G|)} \right]$$

- Haluška (2003)

$$d(F, G, f, g) = \min(f, g) \left[ |F - G| e^{-|F-G|} \right]$$

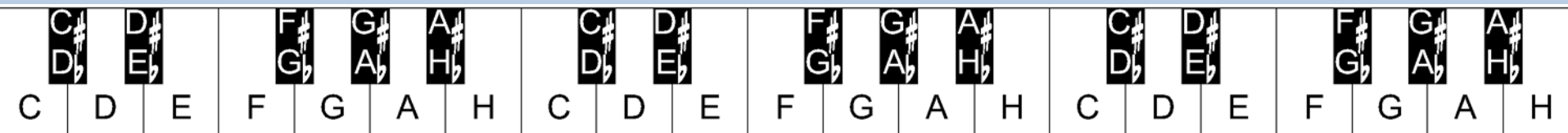
These formulas does not fulfill expected properties



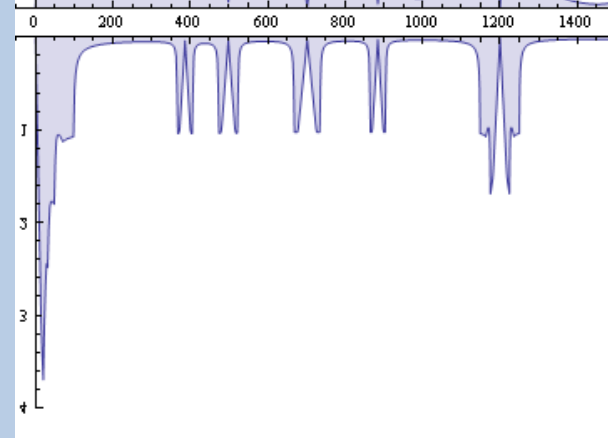
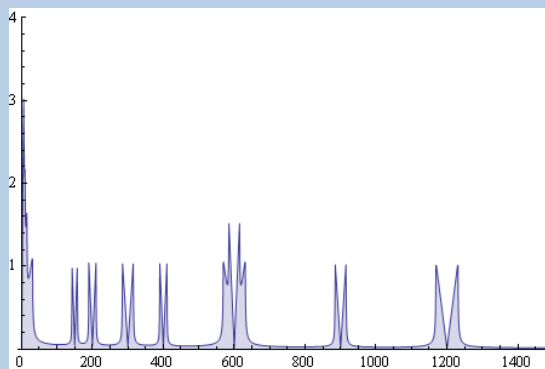
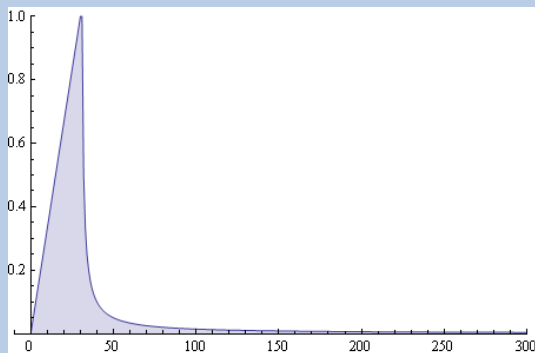
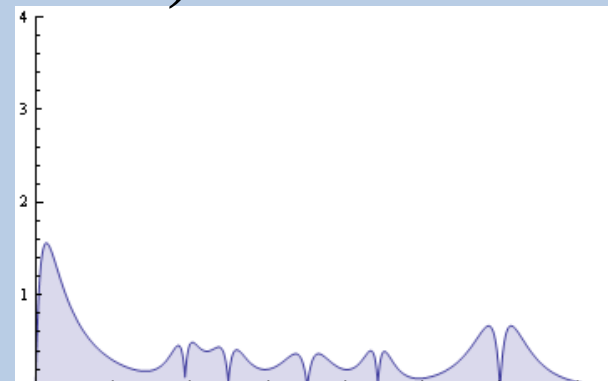
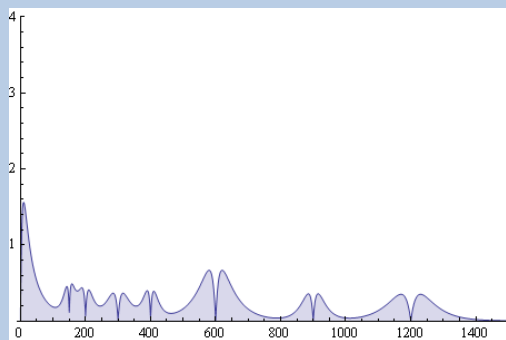
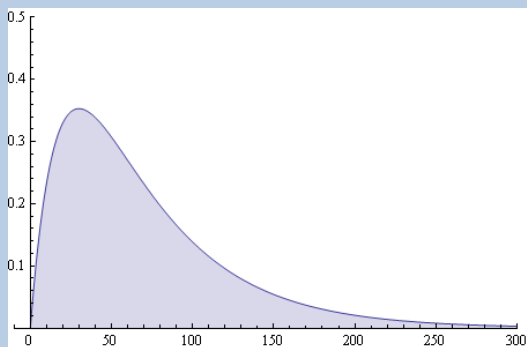


## Our research

$$\begin{aligned}
 & \blacksquare \quad d(F, G, f, g) = a(f, g) \left[ e^{\frac{-|F-G|^k}{(e-1)b^k}} - e^{\frac{-e|F-G|^k}{(e-1)b^k}} \right] \\
 & \blacksquare \quad d(F, G, f, g) = a(f, g) \left\{ \begin{array}{ll} 0 & |F - G| = 0 \\ \frac{|F - G|}{30} & |F - G| < 30 \\ 1 & |F - G| = 30 \\ \frac{1}{|F - G| - 29} & |F - G| > 30 \end{array} \right.
 \end{aligned}$$



## Our research (visualized)

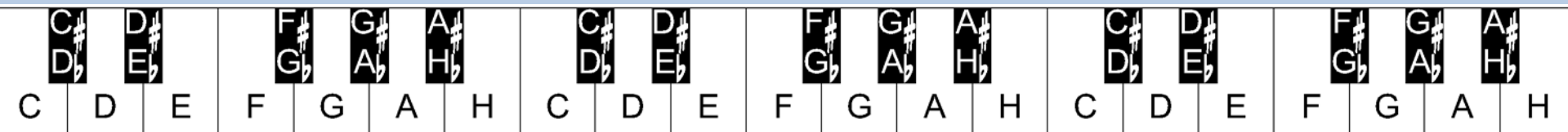


<div>C<sup>#</sup> D<sub>b</sub></div>	<div>D<sup>#</sup> E<sub>b</sub></div>		<div>F<sup>#</sup> G<sub>b</sub></div>	<div>G<sup>#</sup> A<sub>b</sub></div>	<div>A<sup>#</sup> H<sub>b</sub></div>		<div>C<sup>#</sup> D<sub>b</sub></div>	<div>D<sup>#</sup> E<sub>b</sub></div>		<div>F<sup>#</sup> G<sub>b</sub></div>	<div>G<sup>#</sup> A<sub>b</sub></div>	<div>A<sup>#</sup> H<sub>b</sub></div>		<div>C<sup>#</sup> D<sub>b</sub></div>	<div>D<sup>#</sup> E<sub>b</sub></div>		<div>F<sup>#</sup> G<sub>b</sub></div>	<div>G<sup>#</sup> A<sub>b</sub></div>	<div>A<sup>#</sup> H<sub>b</sub></div>	
C	D	E	F	G	A	H	C	D	E	F	G	A	H	C	D	E	F	G	A	H

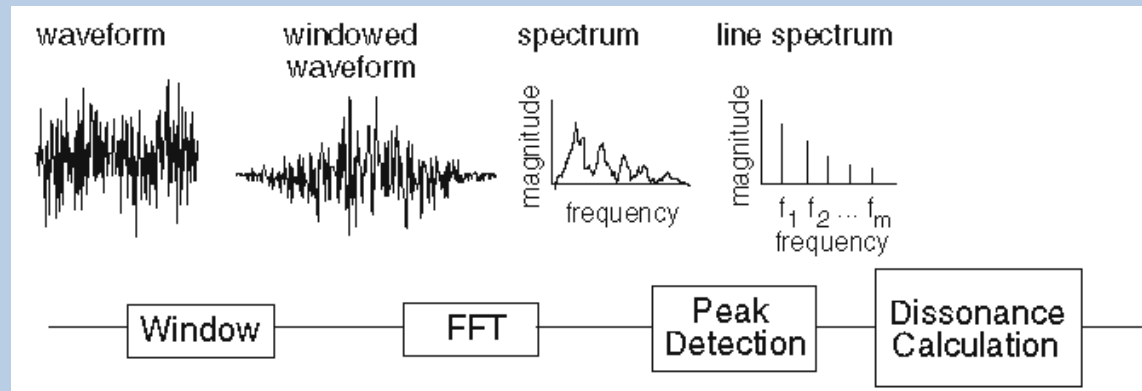
# What it is good for?

- Measure the sensory dissonance of a sound
  - Compare musicians / orchestras
  - Analyze a musical piece
  - Specify historical tunings
- Create a sound with a specific value of dissonance
  - Synthesize tones / create sounds
  - Create new tuning systems
  - Create/specify new instruments

And more...



## Conclusion



C <sup>#</sup> D <sub>b</sub>	D <sup>#</sup> E <sub>b</sub>	F <sup>#</sup> G <sub>b</sub>	G <sup>#</sup> A <sub>b</sub>	A <sup>#</sup> H <sub>b</sub>	C <sup>#</sup> D <sub>b</sub>	D <sup>#</sup> E <sub>b</sub>	F <sup>#</sup> G <sub>b</sub>	G <sup>#</sup> A <sub>b</sub>	A <sup>#</sup> H <sub>b</sub>	C <sup>#</sup> D <sub>b</sub>	D <sup>#</sup> E <sub>b</sub>	F <sup>#</sup> G <sub>b</sub>	G <sup>#</sup> A <sub>b</sub>	A <sup>#</sup> H <sub>b</sub>
C	D	E	F	G	A	H	C	D	E	F	G	A	H	

## Conclusion

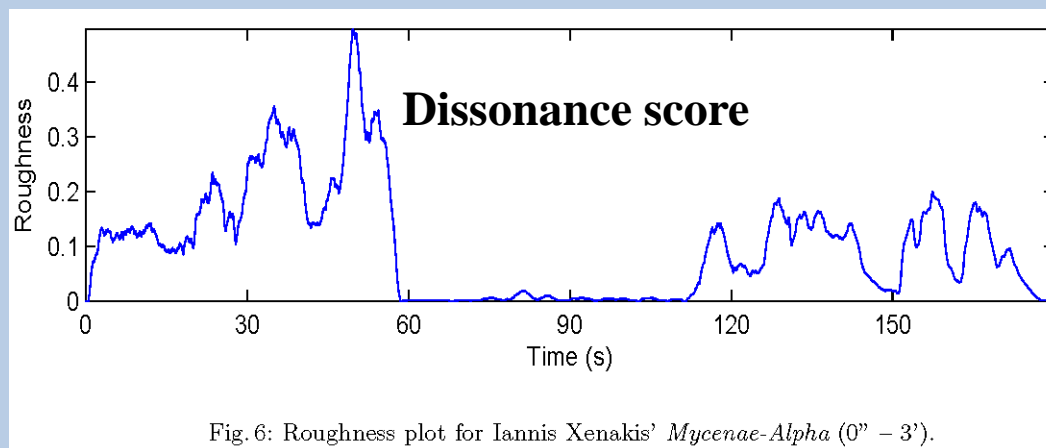
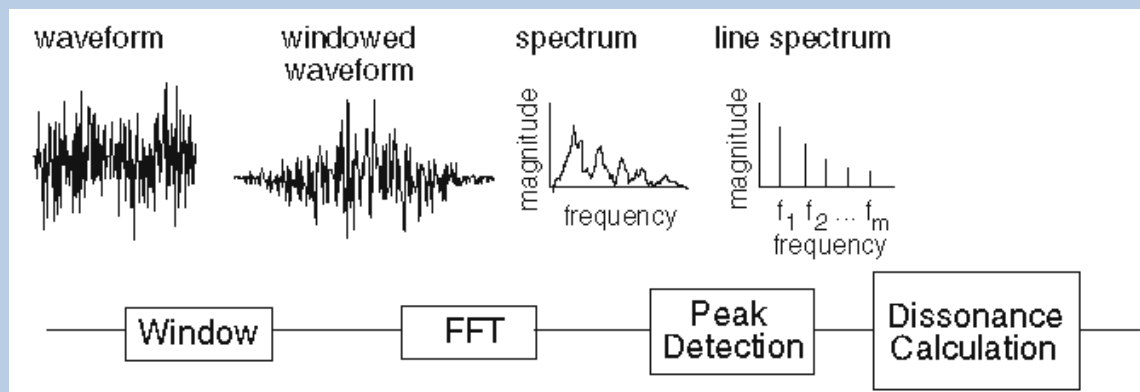


Fig. 6: Roughness plot for Iannis Xenakis' *Mycenae-Alpha* (0'' – 3').

**John MacCallum, Aaron Einbond (2008)**

C <sub>♯</sub>	D <sub>♯</sub>	F <sub>♯</sub>	G <sub>♯</sub>	A <sub>♯</sub>	C <sub>♯</sub>	D <sub>♯</sub>	F <sub>♯</sub>	G <sub>♯</sub>	A <sub>♯</sub>	C <sub>♯</sub>	D <sub>♯</sub>	F <sub>♯</sub>	G <sub>♯</sub>	A <sub>♯</sub>	
D <sub>♭</sub>	E <sub>♭</sub>	G <sub>♭</sub>	A <sub>♭</sub>	H <sub>♭</sub>	D <sub>♭</sub>	E <sub>♭</sub>	G <sub>♭</sub>	A <sub>♭</sub>	H <sub>♭</sub>	D <sub>♭</sub>	E <sub>♭</sub>	G <sub>♭</sub>	A <sub>♭</sub>	H <sub>♭</sub>	
C	D	F	G	A	C	D	F	G	A	C	D	F	G	A	H

# Thank you for your attention

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