

An Introduction to Paraconsistent Mathematics

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A paraconsistent logic is one in which local contradictions do not always imply global absurdity, so that it is possible to study inconsistent structures in a coherent way. As an example, the original ‘naive’ theory of sets is very intuitive, but also famously paradoxical. In a paraconsistent set theory, the idea is that some paradoxes can be taken as theorems. This allows a rethinking of some basic discoveries of twentieth century logic, such as undecidability theorems, as well as new ways to make progress into the twenty-first century.

Following through on this idea, I will discuss the paraconsistent investigation of foundational theories, as well as recent developments in the areas of arithmetic, recursion theory, geometry, real analysis and topology. Focus is on two interrelated themes:

- Obstacles and limitations to working with ‘naive’ theories in a weak logic, such as Curry’s paradox, and techniques for dealing with these problems, such as using a substructural logic
- Novel mathematical objects that can only be proved to exist in an inconsistent (but non-trivial) theory, and their properties

My aim is to provide an accessible overview of a burgeoning field of research, including a realistic assessment of both its strengths and weaknesses, philosophical and mathematical.