On subsequences of uniformly distributed sequences

It is well known that the sequence $\{n\alpha\}$ is uniformly distributed for every irrational α. Here $\{x\}$ stands for the fractional part of a real number x. In fact, it is probably the simplest and most frequent kind of example of a uniformly distributed sequence. As every uniformly distributed sequence is dense in $[0,1]$, given any closed set $F \subset [0,1]$, one can choose a subsequence $(n_k(F))$ of positive integers such that the set of all accumulation points of $\{n_k(F)\,\alpha\}$ is exactly the set F. Taking different F_1 and F_2, the corresponding sequences $n_k(F_1)$ and $n_k(F_2)$ will definitely differ. The question is: How much can the sequences $n_k(F)$ have in common for all possible closed $F \subset [0,1]$? The purpose of this talk is to give in a sense surprising answer to this question.